STATE SPACE NEURAL NETWORKS IN NON-LINEAR ADAPTIVE SYSTEM IDENTIFICATION AND CONTROL

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Abstract: Non-linear control design methods, such as feedback linearisation and output regulation theory, are effective techniques for solving non-linear control problems. However, they assume the exact knowledge of the true model and that the states are completely accessible, which is not always true in practice. The starting point for this paper is to develop a viable practical control strategy by combining the modelling capabilities of state space neural networks with the effectiveness of the output regulation theory. By using input and output measurements and based on Lyapunov stability and non-linear observation theories a stable on-line learning methodology for the network parameters is proposed. A practical implementation for controlling a multivariable process is included to illustrate the effectiveness of the proposed adaptive control methodology.

Keywords: State space neural networks, on-line learning, non-linear control, output regulation.

1. INTRODUCTION

In many real world applications there are non-linearities and unmodelled dynamics which poses problems when implement practical control strategies. Modern control (such as adaptive and optimal control techniques) and classical control theories have been commonly applied to deal with these difficulties. In the application of such techniques the existence of a linear model of the system is a prior assumption. However, such linearised model may not reflect the true physical properties of the system to be controlled.
Neural networks (NN) have attracted a great deal of attention owing to their ability to learn non-linear functions from input-output data examples (Cybenko, 1989). Applied to control field, NN are essentially non-linear models that can be useful to solve non-linear control problems (Narendra, 1996), (Hunt and Zbikowski, 1997).
Basically, NN can be classified as static (feedforward) and dynamic (recurrent) (Tsoi and Back, 1997). Recurrent neural networks (RNN) were first introduced by Hopfield (1982) and then developed by some other authors (Rovithakis and Christodoulou, 1997), (Suykens et al, 1997), (Poznyak et al, 1999), (Kulawski and Brdys, 2000). Due to their intrinsic abilities to incorporate time, RNN have some advantages with respect to static NN for modelling dynamic processes and are used in this work.
Concerning control strategies, there are several ways in which neural models can be used for control purposes (see, for example, the reviewing work of Agarwal, 1997). Although, there are two main ideas: the network is itself a controller or the network just performs modelling tasks, being the controller design performed by using an indirect methodology. This last mentioned strategy is employed in this paper.
The training of a NN can be performed on-line or off-line. Although off-line training is usually straightforward, conditions for assuring good generalization of the NN through out the control space are difficult to obtain, which makes on-line training always necessary in control applications. The idea followed here is to obtain a previous neural network, as a first model approximation using the available data, and applying an on-line learning algorithm to improve the discrepancies between the output of this original model and the actual output of the system. By means of a Lyapunov analysis a stability condition for the weights updating is developed.
One of the challenges in control theory is the tracking and disturbance rejection problems in non-linear systems. This problem is known as the non-linear output regulation (OR) problem, or the servomechanism problem and the main goal is to derive a control law such that the closed loop system is stable and, simultaneously, the tracking output error converges to zero. These results lead to a straightforward method for solving non-linear control problems. However, one drawback of the OR theory is that it assumes a perfect model knowledge and that the system states are completely accessible. First, to mitigate the modelling uncertainties an adaptive control technique is used. Second, if only inputs and outputs are accessible a Luenberger observer (1971) can be employed. The difficulty is that the model based observer cannot be used, because no exact knowledge of the plant is available. Model-free observers, such as high-gain observers (Li et al, 1999) or sliding mode observers (Choi et al, 1990) may be used instead, but are only suitable for particular class of plants.
In this paper the modelling capabilities of a state space NN, to act as an observer for a non-linear process allowing a simultaneous estimation of parameters and states, with the effectiveness and stability properties of OR are combined. The control structure consists in an indirect control scheme and the main goal is to provide a viable practical control strategy to be implemented on general non-linear discrete real-time control problems.

The paper is organised as follows: in section 2 the proposed NN architecture and the associated off-line and on-line learning laws are presented. Section 3 reviews the non-linear OR theory and section 4 presents the adaptive neural-control structure. In section 5 some experimental results on a laboratory process are presented and in section 6 some conclusions are stated.

2. STATE SPACE NEURAL NETWORKS

The process to be controlled is assumed to be described in the form (1)

$$\begin{aligned} x_p(k+1) &= f\,(\,x_p(k),\, u(k)\,) \\ y(k) &= C\,x_p(k) \end{aligned} \tag{1}$$

where $f{:}\Re^{n_p}\times\Re^{n_u}\rightarrow\Re^{n_p}$ is a non-linear function. The vector $x_p\in\Re^{n_p}$ defines the state of the process (assumed to be inaccessible), $u\in\Re^{n_u}$ and $y\in\Re^{n_y}$ are, respectively, the process input and output.

2.1 Proposed architecture

Given the approximation capabilities of RNN (Jin et al, 1999) it is assumed that there exist a RNN, described by (2), that is able to describe the plant's dynamics.

$$\begin{aligned} x_n(k+1) &= A\,x_n(k) + D\,\sigma(\,x_n(k)\,) + B\,u(k) \\ y_n(k) &= C\,x_n(k) \end{aligned} \tag{2}$$

The vector $x_n\in\Re^{n}$ is the output of the hidden layer, known as the network hyper-state, and $y_n\in\Re^{n_y}$ is the network output. $A\in\Re^{n\times n}$, $B\in\Re^{n\times n_u}$, $C\in\Re^{n_y\times n}$, $D\in\Re^{n\times n}$ are interconnection matrices and the neural activation function $\sigma(\cdot)$ is the hyperbolic tangent function. This architecture can be seen as a modification of the original discrete time RNN proposed by Hopfield, with an additional exogenous input. On the other hand, this is a hybrid model, with a linear and a non-linear part.

2.2 Parameter estimation

Off-line learning*:* As pointed out by Hagan and Menhaj (1994) the Levenberg-Marquardt is more efficient than other techniques when the network contains no more than a few hundred parameters. Due to its effectiveness this algorithm has been applied for the off-line training of the RNN. From this initial training phase results the network parameters, expressed in A^*, B^*, C^* and D^* matrices.

On-line learning: Several training algorithms have been proposed to adjust the network parameters in recurrent networks. Typical examples are the real time recurrent algorithm (Williams and Zipser, 1995), the dynamic backpropagation (Narendra and Parthasarathy, 1991), and the backpropagation trough time (Werbos, 1990). Unfortunately few stability studies were addressed. A main contribution of this paper is to propose a stable on-line learning law for the RNN based on the dual Kalman strategy, where both the hyper-state and the parameters are updated.

To this aim it is assumed that the matrices A^* and C^* are static (off-line evaluated) and only the matrices $B(k)$ and $D(k)$ are to be updated on-line. Additionally, it is assume that the hyper-state $x_n(k)$ is unknown and is to be determined by an observing procedure. For this reason the pair (A^*, C^*) is assumed observable.

It is important to stress that, actually, the network does not behave as an observer, in the strictest sense. In fact, it is not expected to estimated correctly the system state x_p (which is assumed unknown), but only one possible representation in state space form for the system. That is, it is assumed that there exist a state space NN (3), with hyper-state x_s , such that the real process output (y) and the neuronal output (y_s) are equivalent.

$$\begin{aligned} x_s(k+1) &= A\,x_s(k) + B\,u(k) + D\,\sigma(\,x_s(k)\,) \\ y_s(k) &= C\,x_s(k) \end{aligned} \tag{3}$$

The state and parameter network identification procedure aims to estimate the neural state x_n and parameters A, B, C, D, such that the estimated state x_n and output y_n converge to the true values of the ideal network model, respectively x_s and y_s.

State observation

The network model is a hybrid one. It has a linear and a non-linear part where the non-linear part is due to the hyperbolic tangent, a Lipschitz non-linear function. Based on this idea the observation results for continuous non-linear systems proposed by Thau (1973), are extended to the proposed recurrent non-linear model (3).

Theorem 1

Consider the state space RNN (3), with the pair (A,C) observable and characterised by a Lipschitz non-linearity (τ is the Lipschitz constant). If there exist a gain matrix $L\in\Re^{n\times ny}$, such that $A_o = A - LC$ is stable and definite positive matrices P and Q verifying the discrete time Lyapunov equation

$$A_o^{\,T} P A_o - P = -Q \tag{4}$$

then the hyper-state updating defined by

$$\begin{aligned} x_n(k+1) &= A\,x_n(k) + B\,u(k) + D\,\sigma(x_n(k)) + L\,\big(y_s(k) - C\,x_n(k)\big) \\ y_n(k) &= C\,x_n(k) \end{aligned} \tag{5}$$

ensures state convergence, i.e.,

$$\lim_{k\to\infty}\; x_n(k) \rightarrow x_s(k) \tag{6}$$

if

$$\|D\,\tau\| \le -\|A_o\| + \sqrt{\|A_o\|^2 + \frac{\lambda_{min}(Q)}{\lambda_{max}(P)}} \tag{7}$$

Theorem 1 provides, like the Thau observer (1973), only a procedure to check the stability of the error after the gain *L* has been evaluated. It does not suggest a method to design a stable observer.

Parameter adaptation

After establishing a estimation procedure for the states, the problem of parameter estimation is addressed. Similar to a dual Kalman filter, the states $x_s(k)$ are assumed to be known and the problem consists in updating the network parameters $B(k)$ and $D(k)$ such that the estimation error, defined by (8), converges to zero.

$$\varepsilon(k) = x_s(k) - x_n(k) \tag{8}$$

Considering the off-line training parameters and using (2) and (3) it is possible to write

$$\begin{aligned}\varepsilon(k+1) = A^* x_s(k) - A\, x_n(k) + (D^* - D(k))\, \sigma(x_s(k)) \\ + (B^* - B(k))\, u(k)\end{aligned} \tag{9}$$

Defining $\tilde{D} = D^* - D(k)$, $\tilde{B} = B^* - B(k)$ and assuming $A = A^*$ results (10).

$$\varepsilon(k+1) = A\, \varepsilon(k) + \varphi(k)\, \vec{\tilde{W}}(k) \tag{10}$$

$\varphi(k) \in \Re^{n \times nw}$ is an information matrix, depending on $\sigma(x_s(k))$ and $u(k)$. $\vec{\tilde{W}}(k) \in \Re^{n_w}$ is the vector of parameters to be estimated,

$$\vec{\tilde{W}}(k) = [\ \vec{\tilde{D}}\,(k) \quad \vec{\tilde{B}}\,(k)\ ] \tag{11}$$

where $n_w = (n \times n + n \times n_u)$ is the number of parameters to be updated. Since

$$\Delta \tilde{W}(k) = \tilde{W}(k) - \tilde{W}(k-1)$$

$$\Delta \tilde{W} \tag{12}$$

$$(k) = (W^* - W(k)) - (W^* - W(k-1)) = -\Delta W(k)$$

and by using (11) it is possible to write (13).

$$\varepsilon(k+1) = A\, \varepsilon(k) - \varphi(k)\, \Delta\vec{W}(k) \tag{13}$$

Based on Lyapunov stability theory a weight adaptation law is introduced. The following theorem states this adaptation strategy.

Theorem 2

Given the state space RNN (3), and the network model (2) with $A = A^*$ and $C = C^*$ the updating law

$$\Delta\vec{W}(k) = M(k)^{-1}\, \varphi(k)^T P A\, \varepsilon(k) \tag{14}$$

ensures the stability and convergence of the estimation error (8) if A is a Hurwitz matrix, $M(k) \in \Re^{n_w \times n_w}$ is given by (15) and $P \in \Re^{n,n}$ is obtained from the discrete time Lyapunov equation (16).

$$M(k) = [\ I + \frac{1}{2}\, \varphi(k)^T P\ \varphi(k)\ ] \tag{15}$$

$$A^T P A - P = -Q \tag{16}$$

$Q \in \Re^{n \times n}$ is a definite positive matrix and I is an identity matrix of appropriate dimensions.

Remarks: Since A is Hurwitz P is unique and always exists (Goodwin et al, 1980). The developed method is only applicable in the case of A is a Hurwitz matrix, i.e., when the linear part of the network model is stable. For this reason this matrix was assumed constant after the off-line training procedure. In practice, such imposition restricts the initial determination of the network parameters and the class of systems to which the method can be applied.

3. NON-LINEAR OUTPUT REGULATION

The OR problem for linear systems was solved by Francis and Wonham (1975) and Francis (1977). For non-linear discrete time systems, Castillo et al, 1993, using the zero output constrained algorithm (Monaco and Normand-Cyrot, 1987), showed that the solution for the problem is reduced to the solution of transcendental non-linear equations, which represent the discrete time counterpart of the differential and transcendental equations, found for the continuous time systems by Isidori and Byrnes (1990).

3.1 Problem description

Given a system in the form (1) to be controlled and considering an additional external variable $w(k)$, the discrete time system is given by (17).

$$\begin{aligned} x(k+1) &= f\left(x(k), u(k, w(k) \right) \\ y(k) &= C\, x(k) \\ w(k+1) &= s\left(w(k) \right) \\ e(k+1) &= h\left(w(k), x(k) \right) \end{aligned} \tag{17}$$

The vector $w \in \Re^{n_y}$ defines the disturbances and/or the reference signal generated by a so-called exosystem, and $e \in \Re^{n_y}$ defines the output tracking error. It is assumed that the mappings $f(x,u,w)$ and $s(w)$ are smooth functions satisfying $f(0,0,0) = 0$ and $s(0) = 0$. Given this extended system, the problem of asymptotically tracking a reference trajectory is to find a state feedback control law such that the error $e(k)$ goes to zero and the whole system is asymptotically stable. More specifically, it is desired to find conditions under a controller in the form (18),

$$u(k) = \gamma\,(\,x(k), w(k)\,) \tag{18}$$

where $\gamma{:}\Re^n \times \Re^{n_y} \to \Re^{n_u}$ is a smooth mapping such that $\gamma(0,0) = 0$, satisfying the following two requirements.

S1: The equilibrium point $x = 0$ of dynamics

$$x(k+1) = f\,(\,x(k), \gamma(x(k),0)\,) \tag{19}$$

is locally exponentially stable.

S2: There exists a neighbourhood of the origin $(0,0)$ such that, for each initial state $(x(0),w(0))$, the solution of the closed loop system (20),

$$\begin{aligned} x(k+1) &= f\,(\,x(k), \gamma(x(k),w(k)), w(k)\,) \\ w(k+1) &= s\,(\,w(k)\,) \end{aligned} \tag{20}$$

satisfies the error condition (21).

$$\lim_{k \to \infty}\ (\,C\,x(k) - r(w(k))\,) = 0 \tag{21}$$

where the desired output (reference) is generated by the exosystem

$$y_d(k) = r(\, w(k)\,) \tag{22}$$

3.2 Problem solution

Theorem (Castillo et al, 1993): The state feedback discrete time regulator problem is locally solvable if there exits two $C^h(h \geq 2)$ mappings $x = \pi(w)$ and $u = c(w)$, with $\pi(w) = 0$ and $c(w) = 0$, satisfying (23).

$$\begin{aligned} \pi(\, s(w)\,) &= f\,(\,\pi(w), c(w), w\,) \\ 0 &= C\,\pi(w) - r(w) \end{aligned} \tag{23}$$

Once evaluated the mappings $x = \pi(w)$ and $u = c(w)$, it is easy to show that the particular control law given by (23), satisfies both requirements S1 and S2.

$$u(k) = \gamma(x,w) = c(w) + K\,(\,x - \pi(w)\,) \tag{24}$$

K is a matrix of appropriate dimensions that places the eigenvalues of the first order approximation of the non-linear state space model in desired locations. As given by equation (24), the solution of the output regulator problem is reduced to a set of non-linear difference equations, known as regulator equations.

3.3 Solution of regulator equations

Except in very few cases, it is difficult to derive an analytical solution to the mappings $x = \pi(w)$ and $u = c(w)$ that solve the regulator equations. One possibility is to solve approximately the regulator equations. Castillo et al, (1993) presented and derived conditions to the existence of an approximate solution for the discrete time case based on a polynomial expansion. Based on a Taylor series expansion as well, (Huang and Rugh, 1992) proposed an approximation method for the continuous case. The same authors presented an alternative approximation (Huang and Rugh, 1999) using a type of RNN, analogous to a cellular network. With a correct choice of parameters, the RNN is able to solve the regulator equations, in the least square sense, by means of a gradient descent minimisation.

Based on a class of RNN, Henriques et al. (2000) have proposed an approximation method to solve the regulator equations. In fact, the regulator equations and the recurrent linear network actually complement each other, i.e, the regulator equations consist in a RNN with the same architecture as the original NN used for modelling purposes. The proposed algorithm leads to a pole placement design ensuring that the solution to the regulator equations converges if the eigenvalues of a given matrix are chosen to be stable.

4. ADAPTIVE CONTROL STRUCTURE

The exact knowledge of the system parameters is not viable in practice. Thus the design of a model based output regulator cannot assure convergence of the tracking error to zero. To overcome this drawback an adaptive strategy is applied providing on-line learning of the neural network's parameters.

The block diagram of the proposed control structure is shown in Fig. 1.

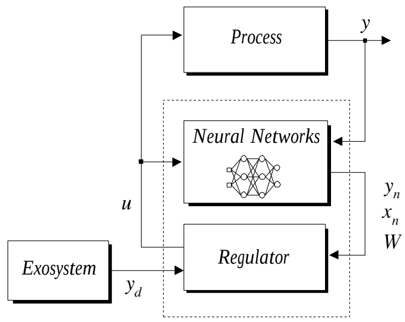


Fig. 1. Proposed control structure.

Based on the identification error, $e_n(k) = y_n(k) - y(k)$ the learning law (section 2) updates the neural parameters, W, and states, x_n. The output regulator design for the NN ensures the asymptotic convergence of the neural tracking error, $e_d(k) = y_n(k) - y_d(k)$. If the parameters of the neural model are adapted in the presence of parametric variations or uncertainties in the dynamics of the system, the system tracking error will converge to zero. In fact, the system tracking error, $e(k) = y(k) - y_d(k)$, can be written as (25)

$$e(k) = y(k) - y_n(k) + y_n(k) - y_d(k) \tag{25}$$

Since the regulator assures (26)

$$\lim_{k \to \infty} (\, y_n(k) - y_d(k)\,) = 0 \tag{26}$$

the overall error will converge provided that the identification error also converges, i.e, (27).

$$\lim_{k \to \infty} (\, y(k) - y_n(k)\,) = 0 \tag{27}$$

5. EXPERIMENTAL RESULTS

In order to illustrate the applicability of the proposed control scheme the liquid level control problem in a laboratory three tank system is considered. The system dynamics is assumed to be totally unknown and the control law is only based on input-output measurements from the plant.

5.1 The laboratory process

The DTS200 three tank system (Amira, 1996) is a multivariable laboratory set-up which consists of three plexiglas tanks interconnected in series by two connecting pipes (see Fig. 2).

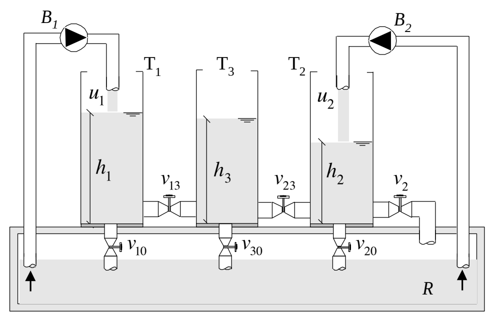


Fig. 2. Schematic diagram of the Laboratory Process.

The liquid leaving T_2 is collected in a reservoir from which pumps B_1 and B_2 supply the tanks T_1 and T_2.

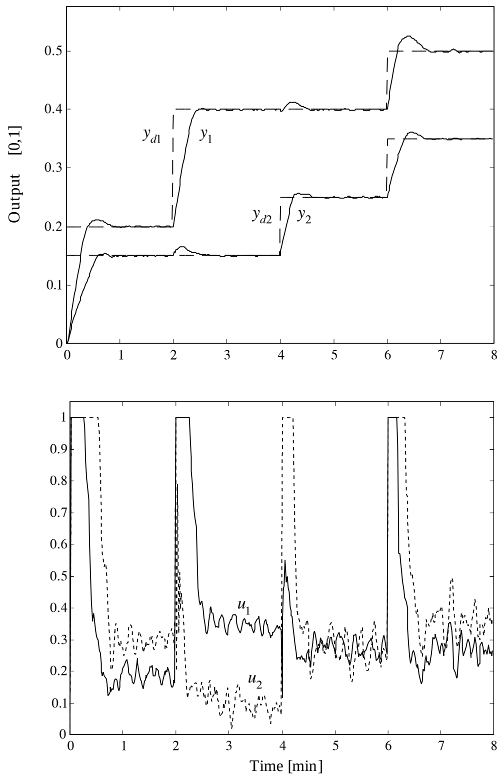
The three tanks are equipped with piezo-resistive pressure transducer for measuring the level of the liquid (usually distilled water). The connecting pipes and the tanks are additionally equipped with manually adjustable valves and outlets for the purpose of simulating clogs as well as leaks. The measured outputs are the water levels y_1 and y_2 (the measurable state variables, h_1, h_2 and h_3, are assume to be inaccessible). The goal of the control system is to control levels y_1 and y_2 by adjusting the flow rate u_1 and u_2, provided by pumps B_1 and B_2, respectively.

5.2 Experiments

Several experiments have been conducted to test the effectiveness, robustness and adaptivity of the NN based controller for the three tanks system. Sampling time was chosen as 1.0 seconds.

Off-line learning: To obtain an initial estimation of the network parameters a number of test inputs have been applied and the resulting outputs were measured. Assuming that the system is non-linear the goal in designing the test inputs was to cover the operational range of the plant to as great an extend as possible. The number of training patterns, hidden neurons, and input sequence are all chosen by experiments since there is still no reliable method of determining these parameters systematically and automatically. It was found that a selection of three hidden neurons, $n = 3$, is suitable to obtain a good model for the laboratory process.

Regulation control problem: The performance of the proposed control scheme was tested for different set points y_d, shown in Fig. 3.



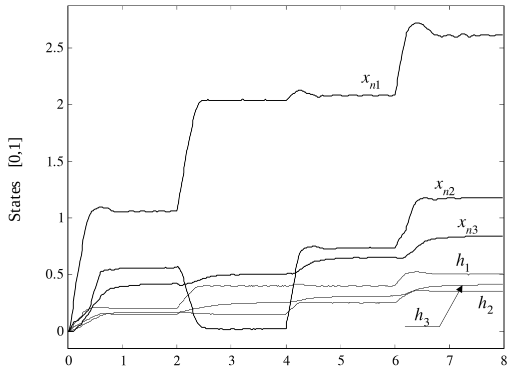


Fig. 3. Controller performance - square wave reference.

As can be observed, the proposed control approach gives quite satisfactory results. In the iterative OR algorithm the eigenvalues were chosen to be placed at 0.8. Although the controller does not use any knowledge of state variables (h_i) it performs reasonable well. As can be seen for this particular experiment in cases where the system states are not accessible the proposed technique seems to be an interesting approach. Thus the difficulties of implement conventional fully-state feedback system can be avoided using this neural control scheme.

Introduction of leaks: Fig. 4 presents experimental results when the connecting pipe ν_{10} from tank T_1 to the reservoir R is partially opened. As can be observed, due to the adaptive characteristic of the proposed method, the control system can handle the dynamic system variation.

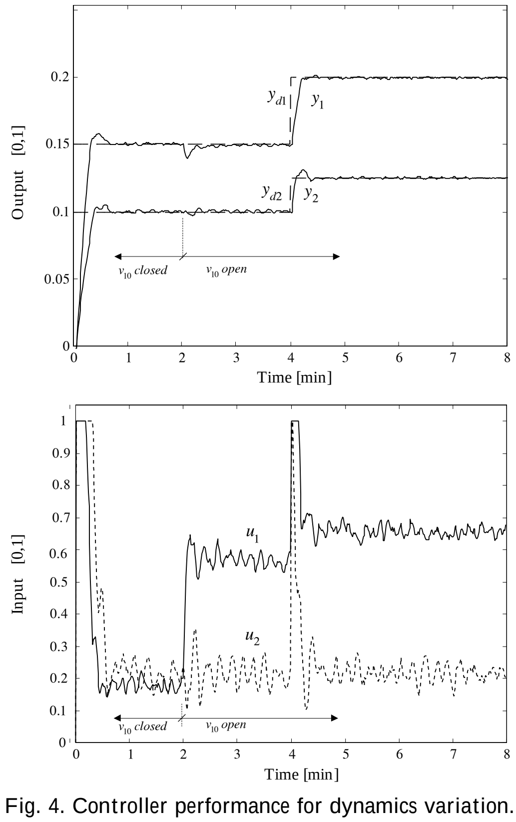


Fig. 4. Controller performance for dynamics variation.

6. CONCLUSIONS

A non-linear control scheme based on a state space neural network has been implement real-time and applied to a multivariable laboratory process. It is a systematic approach which can be easily applied to a wide variety of processes without the use of any initial knowledge of the plant model. To cope with the inaccuracy of the off-line estimated neural parameters, and possible changing dynamics, an adaptive strategy was employed providing an on-line scheme, ensuring stability and convergence properties. In this sense, the neural model can adaptively learn the system uncertainties and the regulator law adjusts the control action in order to guarantee a robust asymptotic error convergence.

This study has shown that neural networks are an important methodology for many industrial control applications. The simplicity and reliability of neuro-control over traditional control techniques will be the key for the development of efficient and intelligent control systems in the near future.

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