A Recurrent Neural Approach for the Nonlinear Discrete Time Output Regulation

Jorge Henriques, B. Castillo-Toledo, André Titli, Paulo Gil and António Dourado

Abstract - **The combination of a recurrent neural network with the output regulation control theory is proposed so that a robust controller for general nonlinear discrete time systems is obtained. It is intended with this approach to profit from the identification capabilities of neural networks with the stability properties of the output regulation theory.**

Given the universal approximation properties, a recurrent neural network is applied for modelling nonlinear systems. Learning is implemented on-line, based on input-output data, ensuring that the learning error converges to zero. Due to the combination of the adaptive neural learning procedure aspect and the output regulator technique the proposed control scheme behaves with strong robustness with respect to unknown dynamics and nonlinear characteristics. To solve the regulator equations a new iterative procedure is presented. The proposed algorithm, based on the recurrent neural network, ensures the convergence of the regulator equations.

Experimental results collected from a laboratory heating system confirm the viability and effectiveness of the proposed methodology.

Index Terms - **Output regulation, recurrent networks, discrete time nonlinear control, on-line learning.**

I. INTRODUCTION

One of the challenges in control theory is the tracking and disturbance rejection problems in nonlinear systems. This problem is known as the nonlinear output regulation problem, or the servomechanism problem. The main goal is to derive a control law such that the closed loop system is stable and, simultaneously, the tracking output error converges to zero. Solving conditions for this problem have been presented, respectively, in continuous and discrete time by [1] and [2].

These results lead to a straightforward method for solving nonlinear output regulation problems. However, one drawback of the output regulation control theory is that it assumes a perfect model. Actually, the control law is no longer valid if there are modelling uncertainties or unmeasured disturbances. One way to mitigate this issue is by using adaptive neuro control techniques.

It has been recognised that neural networks offer potential benefits for application in the field of control engineering, particularly for modelling nonlinear systems [3]. Some of appealing features of neural networks are: their ability for learning through examples, by off-line or on-line training; they do not require any *a priori* knowledge; they can approximate arbitrarily well any nonlinear continuous mapping [4]. Among several architectures found in the literature, recurrent neural networks (RNN), involving dynamic elements and internal feedback connections, have been considered as more suitable for these purposes than feedforward networks. In fact, several authors have shown that recurrent neural networks have a compact structure and they are in a standard form, which makes them an ideal candidate for the application of control synthesis schemes [5].

The main goal of this work is to investigate the combination of the adaptive on line neural learning ability with the output regulator control strategy. The work is organised as follows. In section II a review of output regulation theory is presented, being the problem stated and the proposed control structure presented. Section III and IV contains the main results of the paper. In section III the recurrent neural architecture and the associated off-line and on-line learning laws are presented and, in section IV, the iterative procedure for solving the output regulator equations based on the neural model is described. Section V presents some experimental results, with a laboratory process, and section VI concludes the paper.

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II. OUTPUT REGULATION PROBLEM

A complete solution for linear systems in state space form was given in [6] and [7]. For nonlinear discrete time systems, Castillo [2], using the zero output constrained algorithm [8], shows that the solution for the problem is reduced to the solution of transcendental nonlinear equations, which represent the discrete time counterpart of the differential and transcendental equations, found for the continuous time systems by Isidori, [1]. Moreover, it is proved that their solvability is directly related to the characteristics of the zero dynamics of the system.

A. Problem description

This work considers multivariable systems, with m inputs and p outputs described by state equations of the form (1).

$$x(k+1) = f\{x(k), u(k)\} \quad (1a)$$

$$y(k) = h\{x(k)\} \quad (b)$$

where $f: \Re^{n+m} \to \Re^n$ and $h: \Re^n \to \Re^p$ are nonlinear functions; $u \in \Re^m$, $y \in \Re^p$ and $x \in \Re^n$ are, respectively, the input vector, the output vector and the internal state vector, at a discrete time k. Considering an additional external variable $w(k)$, the discrete time systems (1) is given by (2).

$$x(k+1) = f\{x(k), u(k), w(k)\} \quad (2a)$$

$$y(k) = h\{x(k)\} \quad (b)$$

$$w(k+1) = s\{w(k)\} \quad \text{(c)}$$

$$e(k) = h(x(k)) - r(w(k)) \quad \text{(d)}$$

The vector $w \in \Re^p$ defines the disturbances and/or the reference signal, generated by a so-called *exosystem*, and $e \in \Re^p$ defines the output tracking error. It is assumed that the mappings $f(x,u,w)$, $s(w)$, and $h(x)$ are smooth functions satisfying $f(0,0,0)=0$, $s(0)=0$ and $h(0)=0$. Given this extended system, the problem of asymptotically tracking a reference trajectory is to find a state feedback control law such that the error $e(k)$ goes to zero and the whole system is asymptotically stable. More precisely, it is desired to find conditions under a controller in the form (3),

$$u(k) = ?\{x(k), w(k)\} \quad (3)$$

where $\gamma : \Re^{n+q} \to \Re^m$ is a smooth mapping such that $?(0,0)=0$, satisfying the following two requirements (4) and (5).

S1: The equilibrium point $x=0$ of the dynamics

$$x(k+1) = f\{x(k), g(x(k),0)\} \quad (4)$$

is locally exponentially stable.

S2: There exists a neighbourhood U of the origin $(0,0)$ such that, for each initial state $(x(0), w(0))$, the solution of the closed loop system (5)

$$x(k+1) = f\{x(k), g(x(k), w(k)), w(k)\} \quad \text{(5a)}$$

$$w(k+1) = s\{w(k)\} \quad \text{(b)}$$

satisfies the error condition (6).

$$\lim_{k\to\infty} \{h(x(k)) - r(w(k))\} = 0 \quad (6)$$

B. Problem solution

Theorem [2]: The state feedback discrete time regulator problem is locally solvable if there exits two C^h $(h \geq 2)$ mappings $x = p(w)$ and $u = c(w)$, with $p(w)=0$ and $c(w)=0$, satisfying (7).

$$p(s(w)) = f\{p(w), c(w), w\} \quad \text{(7a)}$$

$$0 = h(p(w)) - r(w) \quad \text{(b)}$$

Once evaluated the mappings $x = p(w)$ and $u = c(w)$, it is easy to show that the particular control law given by (8), satisfies both requirements S1 and S2.

$$u(k) = ?(x,w) = c(w) + K[x - p(w)] \quad (8)$$

K is a matrix of appropriate dimensions that places the eigenvalues of the first order approximation of the nonlinear state space model in desired locations. As given by equation (7), the solution of the output regulator problem is reduced to a set of nonlinear difference equations, known as regulator equations.

C. Regulator equations solution

Except in very few cases, it is difficult to derive an analytical solution for the mappings $x = p(w)$ and $u = c(w)$ that solves the regulator equations. One possibility is to solve approximately the regulator equations. Castillo [2] has presented and derived conditions for the existence of an approximate solution for the discrete time case, based on a polynomial expansion. Based on a Taylor series expansion as well, Huang and Rugh [9] have proposed an approximation method for the continuous case. Recently, the same authors [10] have presented an alternative approximation using a type of recurrent neural network, analogous to a cellular network. With a correct choice of parameters the recurrent neural network is able to solve the regulator equations in the least square sense by means of a gradient descent minimisation.

One of the main contributions of this work is to propose a new iterative algorithm that ensures converge of the solution, as will be presented in section 4.

D. Proposed adaptive control structure

Assuming exact knowledge of the systems parameters is not viable in practice. Thus the design of a model based output regulator cannot assure convergence of the tracking error to zero. Concerning the model, the recurrent neural model can replace the unknown system, transforming the original problem of control into a nonlinear control problem suitable to be designed by the output regulation theory. On the other hand, if the training is performed off-line, the nonlinear neural model is not exact in the presence of disturbances, parameter variations and uncertainties. To overcome this drawback an adaptive strategy is applied providing on-line learning of the network parameters. The block diagram of the proposed control structure is shown if Fig. 1.

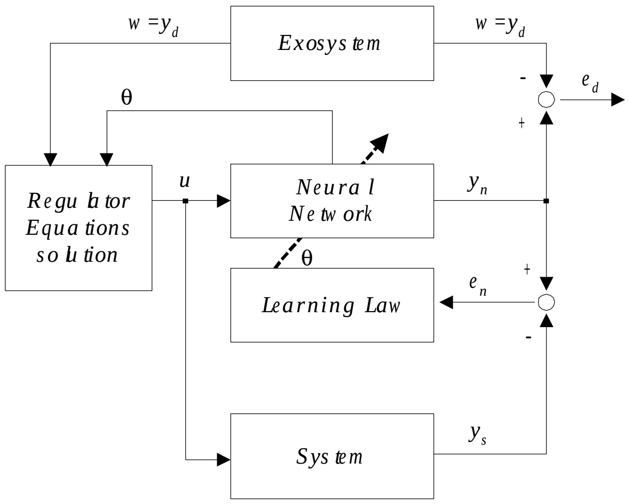

Fig. 1. Proposed control structure.

Based on the identification error, $e_n(k) = y_n(k) - y_s(k)$, a learning law updates the neural parameters, θ. The output regulator design for the neural network ensures the asymptotic convergence of the neural tracking error, $e_d(k) = y_n(k) - y_d(k)$. So, if the parameters of the neural model are adapted in the presence of parametric variations or uncertainties in the dynamics of the system, the system tracking error will converge to zero. In fact, the system tracking error, $e(k) = y_s(k) - y_d(k)$, can be written as

$$e(k) = (y_s(k) - y_n(k)) + (y_n(k) - y_d(k)) \tag{9}$$

Since the regulator assures (10)

$$\lim_{k\to\infty} \{y_n(k) - y_d(k)\} = 0 \tag{10}$$

the overall error will converge provided that the identification error also converges, i.e, (11).

$$\lim_{k\to\infty} \{y_s(k) - y_n(k)\} = 0 \tag{11}$$

III. MODELLING WITH NEURAL NETWORKS

As has been observed from the recent developments of the neural network based control theory, under suitable choice of network architecture, learning methodologies, and appropriate training, neural networks can be conveniently trained to be included in the control system context [3], [11].

A. The proposed architecture

Several discrete time recurrent neural network architectures have been proposed. An excellent reference in this context is the work of Tsoi [12]. The architecture proposed here is depicted in Fig. 2.

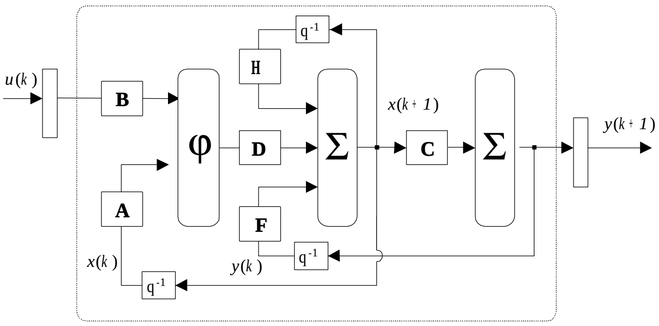

Fig. 2. Block diagram of the recurrent neural network.

Its behaviour can be described by the following discrete time equations (12).

$$x(k+1) = D\varphi\{Ax(k) + Bu(k)\} + H x(k) + Fy(k) \tag{12a}$$

$$y(k) = Cx(k) \tag{c}$$

The vector $x \in \Re^{n}$ is the output of the second hidden layer, $u \in \Re^{m}$ is the input and $y \in \Re^{p}$ is the output. $A \in \Re^{n,n}$, $B \in \Re^{n,m}$, $C \in \Re^{p,n}$, $D \in \Re^{n,n}$ and $H \in \Re^{n,n}$ are the interconnection matrices. This architecture can be seen as a modification of the original discrete time RNN proposed by Hopfield [13]. The neural activation function $\varphi(x)$ is the hyperbolic tangent.

The weights of the network are adjusted to minimise a sum of the squared errors. Once training is performed, the past outputs of the actual system $y_s(k)$ are replaced by the past outputs of the neural network $y_n(k)$ (parallel mode). Following this procedure an autonomous nonlinear state space model (13) is derived, making this architecture well suited for nonlinear control synthesis in which the output regulator strategy is an example.

$$x(k+1) = D\varphi\{Ax(k) + Bu(k)\} + \{H + FC\}x(k) \tag{13a}$$

$$y(k) = Cx(k) \tag{b}$$

B. The proposed learning methodology

The proposed learning methodology consists in a previous off-line learning and a posterior on-line learning.

Off-line learning: As pointed out by Hagan [14] the Levenberg-Marquadt is more efficient than other techniques when the network contains no more than a few hundred of parameters. Due to their effectiveness this algorithm was used for the off-line training of the recurrent neural network.

On-line learning: Several training algorithms have been proposed to adjust the values of the network parameters θ in recurrent networks. Typical examples are the real time recurrent algorithm [15], the dynamic backpropagation [16], and the backpropagation trough time [17]. These methods use a gradient based learning algorithm being the weights changes given by

$$\theta(k+1) = \theta(k) + \Delta\theta(k) \tag{14}$$

with

$$\Delta\theta(k) = \eta(k)P(k)e_n(k) \tag{15}$$

where $e_n(k)$ is the identification error, $\eta(k)$ is a gain parameter and $P(k)$ is function of available information.

In the present work a straightforward variation of this general method was followed. First only the output matrix, C, equation (13a), is updated. With this procedure the dynamics of the neural network, captured with the off-line learning, is fixed and only a gain parameter is updated.

Additionally, at each discrete time step k, an iterative procedure is performed to update the network weights. The increment $\Delta^i\theta(k) = \eta^i(k)P^i(k)e_n^i(k)$, evaluated at each iteration i, is determined such that relation (16) is verified.

$$e_n^i(k) = \alpha\, e_n^{i-1}(k) \tag{16}$$

Following this approach, and under an appropriate selection of the constant α, the identification error converges to zero.

IV. SOLVING REGULATOR EQUATIONS

Due to the nonlinearities, the derivation of an analytical solution for the output regulator problem is, in most cases, impossible. In this work, based on the particular recurrent neural network model, an approximation method is followed. In fact, the regulator equations and the recurrent linear network actually complement each other, i.e, the regulator equations consist in a recurrent neural network, with the same architecture as the original neural network used for modelling purposes.

The proposed algorithm leads the problem to a pole placement design ensuring that the solution of the regulator equations converges if the resulting eigenvalues are chosen to be stable.

A. Algorithm

For a SISO system with n states and $m=p=1$, and tacking into account the relative degree [18], the vector $\Gamma(w)\in\Re^{n+1}$, composed for the mappings that are intended to be determined, is defined by (17).

$$\Gamma(w)=\begin{bmatrix}\Pi_1(w)\\ \Pi_2(w)\\ c(w)\end{bmatrix} \tag{17}$$

where $\Pi_1(w)\in\Re^1$ (the value of the neural network relative degree) and $\Pi_2(w)\in\Re^{n-1}$.

i) Zero output constrained dynamics algorithm

The application of the zero output constrained dynamics algorithm [8] to the present problem leads to the following set of $n+1$ equations with $n+1$ unknowns (18).

i) (18a)

$$C_1\,\Pi_1+C_2\,\Pi_2=w(k)$$

ii) (b)

$$C_1\,j\,\{A_{11}\Pi_1+A_{12}\Pi_2+B_1 g(w)\}+(C_1H_{11}+C_2H_{12})\Pi_1+$$
$$C_2\,j\,\{A_{21}\Pi_1+A_{22}\Pi_2+B_2 g(w)\}+(C_2H_{21}+C_2H_{22})\Pi_2$$
$$=w(k+1)$$

iii) (c)

$$\varphi\{A_{21}\Pi_1+A_{22}\Pi_2+B_2\gamma(w)\}+H_{21}\Pi_1+H_{22}\Pi_2=\Pi_2$$

$C_1\in\Re^{1,1}$, $C_2\in\Re^{1,n-1}$, $B_1\in\Re^{1,1}$ and $B_2\in\Re^{n-1,1}$ are matrices defined respectively by (19) and (20).

$$C=\begin{bmatrix}C_1 & C_2\end{bmatrix} \tag{19}$$

$$B=\begin{bmatrix}B_1\\ B_2\end{bmatrix} \tag{20}$$

The matrices $A_{11}\in\Re^{1,1}$, $A_{12}\in\Re^{1,n-1}$, $A_{21}\in\Re^{n-1,1}$ and $A_{22}\in\Re^{n-1,n-1}$ are defined by (21). H_{ij} has the same dimensions as A_{ij}.

$$A=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix} \tag{21}$$

ii) Setup of the iterative problem

Define an error function, $E\in\Re^{n+1}$, described by (22).

$$E=\begin{Bmatrix}e(1)\\ e(2)\\ e(3)\end{Bmatrix} \tag{22}$$

such that the errors e(1), e(2) and e(3) are given by (23).

$$e(1)=C_1\,\Pi_1+C_2\,\Pi_2-w(k) \tag{23a}$$

$$e(2)=C_1\,j\,\{A_{11}\Pi_1+A_{12}\Pi_2+B_1 g\}+(C_1H_{11}+C_2H_{12})\Pi_1+ \quad \text{(b)}$$
$$C_2\,j\,\{A_{21}\Pi_1+A_{22}\Pi_2+B_2 g\}+(C_2H_{21}+C_2H_{22})\Pi_2$$
$$-w(k+1)$$

$$e(3)=j\,\{A_{21}\Pi_1+A_{22}\Pi_2+B_2 g\}+H_{21}\Pi_1+H_{22}\Pi_2-\Pi_2 \quad \text{(c)}$$

It is possible to define a difference error, between two consecutive iterations, i and i-1, as follows:

$$E^i-E^{i-1}=\begin{Bmatrix}e^i(1) & e^{i-1}(1)\\ e^i(2)- & e^{i-1}(2)\\ e^i(3) & e^{i-1}(3)\end{Bmatrix} \tag{24}$$

Define also the increment between two consecutive iterations given by (25), or (26).

$$\begin{bmatrix}\Pi_1^i(w)\\ \Pi_2^i(w)\\ \gamma^i(w)\end{bmatrix}=\begin{bmatrix}\Pi_1^{i-1}(w)\\ \Pi_2^{i-1}(w)\\ \gamma^{i-1}(w)\end{bmatrix}+\begin{bmatrix}\Delta\Pi_1^i(w)\\ \Delta\Pi_2^i(w)\\ \Delta\gamma^i(w)\end{bmatrix} \tag{25}$$

$$\Gamma^i=\Gamma^{i-1}+\Delta\Gamma^i \tag{26}$$

where Π_1^i, Π_2^i and γ^i and Π_1^{i-1}, Π_2^{i-1} and γ^{i-1} are respectively the values of the mappings to be evaluated at iterations i and i-1.

iii) Convergence of the iterative algorithm

The problem under investigation can be stated as the determination of the increment such that between two consecutives iterations the following condition (27) holds.

$$E^i=\Lambda\,E^{i-1} \tag{27}$$

being $\Lambda\in\Re^{n+1,n+1}$ a matrix with stable eigenvalues. If this relation is verified the error will converge and the regulator equations will have a solution.

iv) Determination of the increment at each time step

The hyperbolic activation function considered in the neural model has some useful properties. Being a non-decreasing monotonic function it is possible to write (28).

$$j(x+\Delta x)=j(x)+k\,\Delta x \tag{28}$$

where k is a value between (x) and $(x+\Delta x)$. The value of k can be approximated by taken a first order approximation

$$j(x+\Delta x)-j(x)=J\Delta x \tag{29}$$

with $J=\dfrac{\partial j(x)}{\partial x}$.

Using (26) and applying this result to equations (23) it is possible to write the difference error as (30)

$$E^i-E^{i-1}=-M\begin{bmatrix}\Delta\Pi^i(w)\\ \Delta\gamma^i(w)\end{bmatrix}=-M\Delta\Gamma \tag{30}$$

where $M\in\Re^{n+1,n+1}$ is a matrix determined by considering the approximation given according to (29).

v) Conclusion

Using this procedure the regulator equations can be set as a problem of pole placement design. In fact, if the increment $\Delta\Gamma$ is chosen as a function of the previous error E^{i-1},

$$\Delta\Gamma = K\, E^{i-1} \tag{31}$$

then

$$E^{i} = \left[I - KM\right] E^{i-1} \tag{32}$$

and, finally, K should be determined such that the eigenvalues of the error equation are in desired locations, that is,

$$eig\left[I - KM\right] = eig(\Lambda) \tag{33}$$

Hence, by a suitable choice of K, the iterative procedure drives the error to zero.

V. EXAMPLE

In order to illustrate the applicability of the proposed control scheme the temperature control in a laboratory heating system is considered. The system dynamics is assumed to be totally unknown and the control law is only based on measurements from the system.

A. The laboratory process

In the laboratory process, depicted in Fig. 3 and Fig. 4, air is forced to circulate by a fan blower through a duct and is heated at its inlet. This is a non-linear process with a pure time delay, which depends on the position of the temperature sensor element and the air flow rate, depending on the damper position Ω. The system input $u(k)$, is the voltage on the heating device, which consists of a mesh of resistor wires, and the output, $y_s(k)$, is the outlet air temperature.

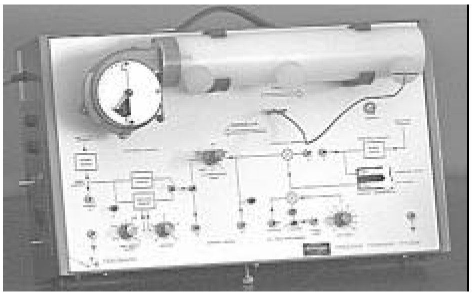

Fig. 3. Laboratory Process.

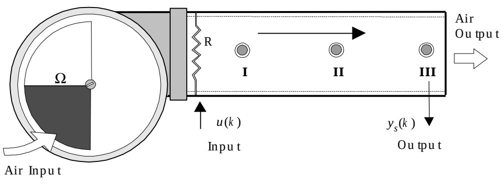

Fig. 4. Schematic diagram of the Laboratory Process.

B. Experiments

Sampling time was chosen as 0.2 seconds. For modelling purposes a recurrent neural network with five neurones in both hidden unit, $n = 5$, was considered. In the iterative output regulation algorithm the eigenvalues were chosen to be placed at 0.2. The recurrent network was previous trained with a representative sequence of data points with the Levenberg-Marquadt algorithm.

The performance of the proposed control scheme was tested for different set points $r(k)$, such as a square wave and a sinusoidal wave. As can be observed in Fig. 5 and in Fig. 6, the proposed control approach gives quite satisfactory results. Clearly it performs considerably well in tracking the set points.

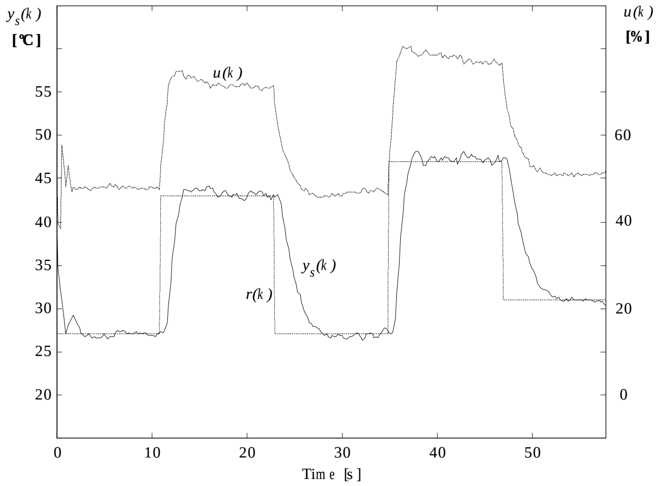

Fig. 5. Controller performance - square wave reference.

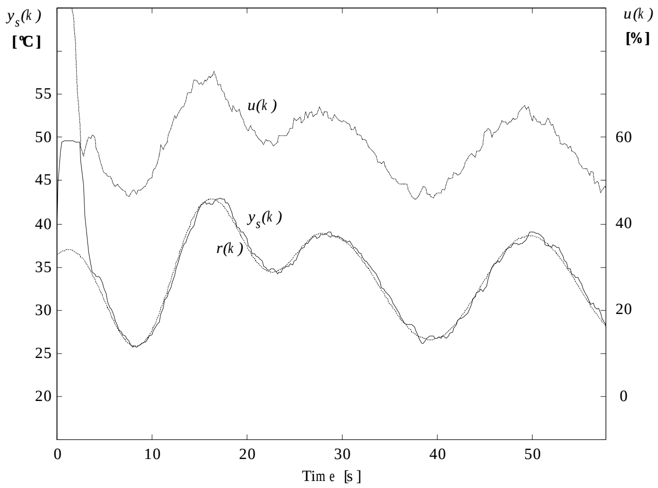

Fig. 6. Controller performance - sinusoidal reference.

Dynamics variation

This experiment addressed the dynamics variation of the laboratory process. Variations were considered in the flow rate of the input air: $\Delta\Omega = +20\%$ at instant 15 seconds and $\Delta\Omega = -20\%$ º, at instant 25 seconds. The reference is a constant value. Fig. 7 presents the experimental results. As can be observed, due to the adaptive characteristic, the control can handle system variations.

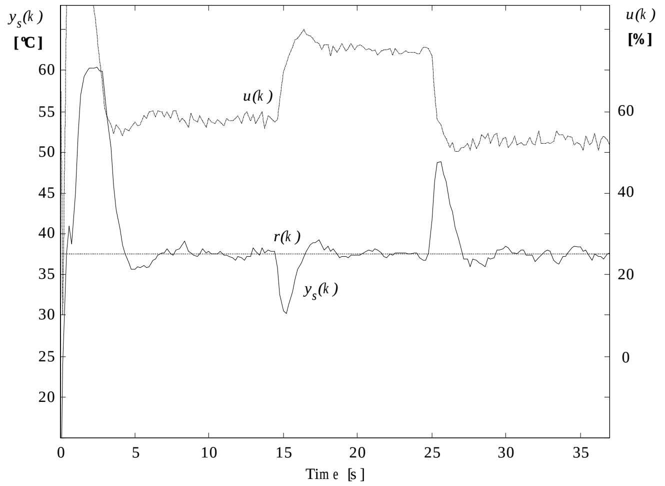

Fig. 7. Controller performance for dynamics variation.

VI. CONCLUSIONS

In this work the combination of the output regulator control theory with recurrent neural networks models was investigated.

The output regulator based on the neural parameters, guarantees that the neural tracking error converges to a small neighbourhood of zero, while the closed loop system is stable. To cope with the inaccuracy of the off-line estimated neural parameters, and possible changing dynamics, an adaptive strategy was employed providing on-line learning of the network parameters. In this sense, the neural model can adaptively learn the system uncertainties and the regulator law adjusts the control action in order to guarantee a robust asymptotic error convergence.

Based on the recurrent neural model an approximated iterative procedure for solving the output regulator design was derived, ensuring convergence of the solution.

The effectiveness of this technique has been demonstrated by several experiments in a laboratory process. Further research will be done towards the study of stability and robustness properties of the proposed control structure.

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