Local Search for the Bi-objective Unconstrained Optimization Problem

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1 Introduction

This work reports an experimental analysis on the properties of graph induced by the set of efficient solutions for a multiobjective combinatorial optimization problem. In particular, we empirically examine *connectedness* of efficient solutions with respect to a given definition of neighborhood and we design methodological principles that exploit this property in order to develop more efficient metaheuristic approaches.

The multiobjective combinatorial optimization problem under investigation is referred as the *Bi-objective Unconstrained Optimization Problem* (BUOP) [1]. Given a set of $i = 1, \ldots, n$ items, each one being associated to a positive profit value p_i and a positive weight value w_i, the BUOP is defined as follows.

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} p_i x_i \\ \min \quad & \sum_{i=1}^{n} w_i x_i \\ \text{subject to: } & x_i \in \{0,1\}^n \end{aligned} \tag{BUOP}$$

The problem can also be seen as a knapsack problem where the capacity constraint is transformed into an additional objective function to be minimized, giving rise to a bi-objective formulation.

2 Connectedness for the BUOP

When trying to identify the whole set of non-dominated solutions, local search algorithms may take advantage of the structure of the efficient set. The issue of *connectedness* is related to the property that efficient solutions are connected with respect to a neighborhood structure [2, 4]. When such a property holds, finding all the efficient solutions is made possible by means of the iterative exploration of the neighborhood of the current approximation by starting with one or more efficient solution(s) [5]. Unfortunately, rather negative theoretical results have been reported in the literature so far with respect to different definitions of neighborhood [2, 4]. However, in practice, empirical results show that approximate non-dominated solutions are strongly clustered for some classical multiobjective combinatorial optimization problems [6].

Here, we investigate connectedness for the BUOP. In order to avoid the use of approximate efficient solutions found by some heuristics approaches (and then biased by the explicit or implicit use of a given neighborhood structure), we implemented an exact algorithm based on dynamic programming for the identification of the maximal complete efficient set. Despite the counterexample reported in [4], practical instances show that connectedness with respect to Hamming distance one holds quite often for problems of different sizes, ranges and objective correlation. In the few cases of unconnectedness with respect to Hamming distance one, we found that connectedness holds for Hamming distance two. These findings give a strong motivation for the exploitation of local search principles in the identification of the efficient set (or its representative) for the BUOP.

3 Pareto Local Search for the BUOP

The proposed approach is a two-phase algorithm based on the Pareto Local Search (PLS) general framework [5]. An archive of mutually non-dominated solutions is maintained. At each iteration, a solution is selected from the archive and its neighbors are explored and used to update the archive. The process stops once the neighborhood of every solution from the archive has been evaluated. The algorithm starts, in the first phase, with a subset of efficient solutions. In the second phase, the archive collects additional non-dominated solutions by the iterative exploration of its neighborhood. As a consequence, if the efficient set is connected with respect to the neighborhood operator under consideration, the proposed approach is able to find the whole set of efficient solutions (even if it may take exponential time if the size of the efficient set is exponential in the size of the problem instance).

For the particular case of the BUOP, a single efficient solution is used to initialize the archive. It consists of the empty solution, that is $x_i = 0, i = 1, \dots, n$. This solution maps, in the objective space, to an extreme point from the non-dominated set, with null profit and weight. A neighborhood operator based on Hamming distance one is considered in the second phase. A given neighbor of a solution can be reached by adding or removing an item. In order to make the computation more efficient, a preparation phase is initially performed in order to rank the set of items into different fronts, with respect to the dominance-depth scheme [3]. Hence, the neighborhood exploration of a given solution is performed in a deterministic order and avoids the evaluation of neighbors that lead to dominated solutions. Experiments are performed on problem instances of different structures and sizes from 300 up to 3000 items, with a 300 increment. As induced by the empirical evidence of connectedness, our results show that the PLS approach leads to the exhaustive identification of a representative set of efficient solutions in most of the cases. Furthermore, it performs very efficiently in comparison to the dynamic programming approach with regards to computational time.

4 Conclusion and Future Work

In this paper, we first studied the connectedness of efficient solutions for the BUOP. Our experimental results suggest that the efficient set is connected in most of the case with respect to Hamming distance one. As a consequence, we use a Pareto local search approach that benefits from this property. It starts with a solution from the efficient set and collects additional non-dominated solutions by means of local search principles. In future works, we plan to investigate connectedness for additional problems, including BUOP formulations with a cardinality constraint and with multiple choices, as well as to design efficient PLS-like algorithms for their resolution.

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