

# COMBINING A RECURRENT NEURAL NETWORK AND THE OUTPUT REGULATION THEORY FOR NON-LINEAR ADAPTIVE CONTROL

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## Abstract

In this paper an affine recurrent neural network is combined with the output regulation theory, providing a model based control scheme for non-linear discrete time systems. Non-linear control design techniques can be successfully applied for non-linear plants. However, they assume the knowledge of a mathematical model and that the states are completely accessible, which is not always true. Due to their approximation capabilities, their ability for on-line learning and their analogy with state space models, recurrent neural networks can be effectively employed for modeling purposes. An affine recurrent neural network is considered, being the on-line estimation of parameters addressed taking into account the Lyapunov stability theory. Motivated by their stability properties, the output regulation theory is considered here for control design, in an adaptive framework. However, the derivation of an analytical solution for the output regulation problem is not viable in practice. The main contribution of this work is to propose a strategy, based a particular recurrent neural model, that leads the regulation problem to a pole assignment solution ensuring the regulator equations convergence. To illustrate the applicability of the proposed adaptive control scheme the concentration control of a benchmark continuous tank reactor is considered.

**Keywords:** Output regulation, recurrent neural networks, discrete-time non-linear control, CSTR.

## 1. INTRODUCTION

The evolution that has been verified in information technologies has provided tools for the development and practical application of new theories and methods. Nowadays, it is possible to develop sophisticated algorithms, providing new perspectives and challenges in several engineering fields, namely in the automatic control area. The soft computing concept has emerged as an association of several computation methodologies, such as neural networks and fuzzy systems with capabilities to provide the basis for the conception and design of intelligent systems that can exploit the tolerance to imprecision and uncertainty, learn from experience and adapt to changes in the operating conditions [1].

At the same time, there have been significant developments in conventional theory area. These developments have naturally originated some competition, which have provided new opportunities for the automatic control evolution. At present one can observe several efforts in order to establish bridges among these paradigms as well as synthesizing methodologies and control schemes that merge these new tools in order to establish a common framework. The approach in this paper follows this direction and the purpose consists into profiting from the identification capabilities of neural networks and from the stability properties of the output regulation theory.

In many real world applications there are non-linearities and unmodelled dynamics that cause problems in implementation of control strategies. Following a conventional approach the design of a controller usually demands a precise analytical model of the process under consideration. However, processes are often very complex making impossible to derive a useful analytical model. In this case the problem of developing an experimental mathematical model based on observed input and output data has attracted much attention. The ability of neural network models to learn and generalize based on input-output behavior of a process has had a great impact in this context. Its universal approximation properties, generality and its ability to adjust on-line its parameters using data examples, has allowed neural networks to answer two of the main challenges for which conventional techniques have present some difficulties: generality and precision in modeling problems and adaptation abilities to time-variant processes [2].

Basically, neural networks can be classified as static (feedforward) and dynamic (recurrent). Due to their intrinsic ability to incorporate time, recurrent neural structures have advantages with respect to static neural networks for modeling dynamic processes and are used in this work. Furthermore, they are in a standard form and they present a lower order compact structure, which makes them ideal candidates for implementation in model based adaptive control schemes. However, very few structures have been proposed specially for control purposes. The *affine* structure proposed in this work can be seen as an attempt to establish a compromise between two main goals: the development of generic structures, that can be used for modeling non-linear system but, at the same time, being effective for both parameter estimation and control design procedures. Concerning control strategies there are several ways in which neural models can be used as the basis for control design, [3]. There are two main ideas: the first one, where the controller is itself a neural network (in this case the most common strategy consists on learning the plant's inverse); the second, where the networks just perform modeling tasks being the controller design carried out through a classical control technique. Though the use of a neural network to learn the plant inverse is theoretically a viable option, in practice the obtained inverse model may be not well defined or stable. Mainly for this reason the second mentioned control strategy, analogous to a conventional indirect strategy, is the approach followed in this paper. Thus, assuming their approximation properties [4], the recurrent neural model can replace the unknown system, resulting in a non-linear control problem suitable to be designed by non-linear control techniques.

The selection of the structure and learning procedure are not addressed here. It is assumed that the neural model is well-defined and an appropriate on-line method for adjusting its parameters is

available, so the only problem that is under consideration in this paper concerns the control design, based on a non-linear neural model. In this context the geometric approach has provided a variety of tools for the analysis and design of control systems, where the most common is the feedback linearisation technique. Basically, this technique transforms the original non-linear input-output process into one which is linear in the new transformed input. The use of this transformation is, however, restricted to minimum phase systems and systems with an invertible characteristic matrix.

The main goal of output regulation (*OR*) is to derive a control law such that the closed loop system is stable and, simultaneously, the tracking output error converges to zero leading to a straightforward method for solving non-linear control problems [5]. However, due to the non-linearities, the derivation of an analytical solution to the output regulator problem is, in most cases, impossible. In this work, based on the particular *RNN* model, an approximation method is proposed. This approach leads the problem to a pole placement design ensuring the solution of the regulator equations converges if the resulting eigenvalues are chosen to be stable.

Chemical systems are often highly non-linear and so difficult to control. A well-known example is the continuous stirred tank reactor (*CSTR*) benchmark. The common used model for this process has few state variables but the controller design is highly involved due to the non-linear characteristics of the process. Thus, the *CSTR* process is a typical example for which the conventional linear control techniques presents some difficulties and is considered in this work as a means for assessing the viability of the proposed non-linear neuro-control technique. The paper is organized as follows. In section 2 the problem is stated: the recurrent neural architecture and the associated learning law are presented as well as the adaptive neuro-control approach. Section 3 reviews the non-linear output regulation control theory and the proposed iterative procedure for solving the output regulator equations is described. Section 4 presents some simulation results for the *CSTR* process and section 5 concludes the paper.

## 2. PROBLEM STATEMENT

This work considers multivariable systems, with  $n_u$  inputs and  $n_y$  outputs described in the form (1).

$$\begin{aligned} x_p(k+1) &= f(x_p(k), u(k)) \\ y(k) &= C x_p(k) \end{aligned} \quad (1)$$

$f: \mathfrak{R}^{n_p \times n_u} \rightarrow \mathfrak{R}^{n_p}$  is a non-linear function, the vector  $x_p \in \mathfrak{R}^{n_p}$  defines the state of the process (assumed to be unknown),  $u \in \mathfrak{R}^{n_u}$  and  $y \in \mathfrak{R}^{n_y}$  are, respectively, the process input and output, at a discrete time  $k$ .

### 2.1 Modeling With Recurrent Neural Networks

Given the approximation capabilities of *RNN* it is assumed that there exist an *affine RNN*, described by (2), able to describe the plant input-output dynamics (1).

$$\begin{aligned} x_n(k+1) &= A x_n(k) + D \sigma(x_n(k)) + B u(k) \\ y_n(k) &= C x_n(k) \end{aligned} \quad (2)$$

The vector  $x_n \in \mathfrak{R}^n$  (known as hyper-state) is the output of the hidden layer and  $A \in \mathfrak{R}^{n,n}$ ,  $B \in \mathfrak{R}^{n,n_u}$ ,  $C \in \mathfrak{R}^{n_y,n}$ ,  $D \in \mathfrak{R}^{n,n}$  are the interconnection matrices. This architecture can be seen as a modification of the original *RNN* proposed by Hopfield [6], with an additional exogenous input and a linear part. The activation function  $\sigma(x)$  is the hyperbolic tangent function.

The learning methodology consists in a previous off-line learning and a posterior on-line adjusting parameters procedure. Concerning the off-line learning the Levenberg-Marquardt was used. As it has been pointed out by Hagan [7] the Levenberg-Marquardt algorithm is more efficient than other techniques when the network contains no more than a few hundred of parameters.

Several training algorithms have been proposed to adjust on-line *RNN* weights, such as the backpropagation trough time [8], the dynamic backpropagation [9], and the real time recurrent algorithm [10]. Basically, these methods use a gradient based algorithm where the weights  $W \in \mathfrak{R}^{n_w}$  ( $n_w$  is the number of parameters) are updated by (3).

$$W(k+1) = W(k) + \Delta W(k) \quad (3)$$

However, with respect to the updating law few stability studies have been presented. In [11]- a stable on-line learning law for the *RNN* based on a dual Kalman strategy is proposed. Based on Lyapunov stability and non-linear observation theories both the hyper-state  $x_n$  and the parameters  $W$  are updated providing stability and convergence of the identification error  $e_y(k)$

$$\lim_{k \rightarrow \infty} [y(k) - y_n(k)] = e_y(k) = 0 \quad (4)$$

In this approach it is assumed that the eigenvalues of the matrix  $A$  lie in the unitary circle and the pair  $(A, C)$  is observable. Moreover, the matrices  $A$  and  $C$  are constant (off-line evaluated) and only the matrices  $B(k)$  and  $D(k)$  are to be updated on-line by the following updating law:

$$\Delta W(k) = \Phi(k)^{-1} \varphi(k)^T P A C e_y(k) \quad (5)$$

where  $\Phi(k) \in \mathfrak{R}^{n_w, n_w}$  is defined by (6) and  $P \in \mathfrak{R}^{n,n}$  results from the discrete time Lyapunov equation (7), where  $Q \in \mathfrak{R}^{n,n}$  is a positive definite matrix and  $I$  is an identity matrix of appropriate dimensions.

$$\Phi(k) = [I + \frac{1}{2} \varphi(k)^T P \varphi(k)] \quad (6)$$

$$A^T P A - P = -Q \quad (7)$$

### 2.2 Non-linear Control Design Approach

The main goal of any controller is to assure that the tracking error  $e_c(k)$ , defined by the difference between the reference  $y_d(k)$  and the actual process output  $y(k)$ , converges to zero. Assuming the separation principle and since the system tracking error can be written as

$$e_c(k) = y(k) - y_n(k) + y_n(k) - y_d(k) \quad (8)$$

and taking into account that the updating process assures the identification error convergence, the overall error will converge, provided the regulator assures that the control error  $e_n(k)$ , defined for the neural model, also converges, that is (9) holds.

$$\lim_{k \rightarrow \infty} [y_n(k) - y_d(k)] = e_n(k) = 0 \quad (9)$$

Most of the stabilization results relies on the Lyapunov theory to develop a stable control law. A distinct approach consists in the output regulation theory, based in the center manifold theory (CMT), [13]. Applied to the *affine RNN* (2) the output regulator design ensures (under some constraints to be presented) the asymptotic convergence of the neural tracking error,  $e_n(k)$ , thus guaranteeing the stability and convergence of the closed loop system

### 3. OUTPUT REGULATION

The output regulation problem for linear systems was solved by Francis and Wonham [14]. For non-linear systems, the fundamental results were introduced by Isidori and Byrnes [5]. They have extended the results of Francis to the general case of a time-varying exosystem, and have shown that the linear matrix equations of Francis are particular cases of certain non-linear partial differential equations, known as *Francis-Isidori-Byrnes* equations. Basically the idea of Isidori and Byrnes consists in finding a feedback control law such that it stabilizes the plant around the equilibrium point at the origin, with a zero exogenous signal (reference). Then, employing the CMT and for a small exogenous signal, the closed loop system will yield asymptotic stability. Isidori and Byrnes have also shown that the solvability of the regulator equations can be expressed in terms of the properties of the zero dynamics of the extended system.

In the discrete-time context, a first attempt to solve the regulation problem was performed by Castillo and Di Gennaro [15] and their results are only applicable for SISO systems. For non-linear discrete time MIMO systems, Castillo *et al* [16], using the Zero Output Constrained Dynamics Algorithm, introduced by Monaco and Normand-Cyrot [17], have shown that the solution to the problem can be reduced to the solution of a non-linear difference system equation, and their solvability is proved to hold under suitable properties for the linear approximation of the system and exosystem. Their results are analogous to the results of Isidori and Byrnes [5], showing in a similar way that the solution of the problem is directly related to the properties of the zero dynamics (eigenvalues do not lie on the unitary circle).

Castillo *et al* [18] have extended their results for non-linear continuous systems with more inputs than outputs. Sureshbabu and Rugh [19] have investigated the output regulation without an exosystem assumption and further pursue the advantages of using both the exogenous signal and its time derivative in the control law. The same authors [20] have extended their results for the discrete-time setting where current and past values of the exogenous system are considered to be used in the control law. They have formulated an approximate output problem and characterized their solvability in terms of the existence of a certain invariant manifold. Saberi and Stoorvogel [21], for both continuous and discrete-time systems, have proposed a generic formulation for the output regulation problem. Namely, they generalize the problem in order to expand the class of reference and disturbance signals, employing the derivative or feedforward information of the reference signal. However, their results are only applicable to linear systems.

### 3.1 Problem Formulation

Given a discrete time system (1) and considering an additional external variable  $\rho(k)$ , an extended systems can be defined:

$$\begin{aligned} x(k+1) &= f(x(k), u(k), \rho(k)), \quad y(k) = Cx(k) \\ \rho(k+1) &= s(\rho(k)) \\ e(k+1) &= h(\rho(k), x(k)) \end{aligned} \quad (10)$$

The vector  $\rho \in \mathcal{R}^n$  defines the disturbances and/or the reference signal, generated by a so-called exosystem, and  $e \in \mathcal{R}^m$  defines the output tracking error. It is assumed that the mappings  $f(x, u, \rho)$  and  $s(\rho)$  are smooth functions satisfying  $f(0, 0, 0) = 0$  and  $s(0) = 0$ . Given this extended system, the problem of asymptotically tracking a reference trajectory is to find a state feedback control law

$$u(k) = \gamma(x(k), \rho(k)) \quad (11)$$

such that the error  $e(k)$  goes to zero and the whole system is asymptotically stable.

The state feedback discrete time regulator problem is locally solvable, [16], if there exists two mappings  $x = \pi(\rho)$  and  $u = c(\rho)$ , satisfying (12).

$$\begin{aligned} \pi(s(\rho)) &= f(\pi(\rho), c(\rho), \rho) \\ 0 &= C\pi(\rho) - r(\rho) \end{aligned} \quad (12)$$

Once evaluated, the mappings  $x = \pi(\rho)$  and  $u = c(\rho)$ , it is easy to show [5] that a particular control law of (11) is given by (13).

$$u(k) = \gamma(x, \rho) = c(\rho) + K(x - \pi(\rho)) \quad (13)$$

$K \in \mathcal{R}^{n \times n}$  is a matrix that places the eigenvalues of the first approximation of the non-linear state space model ( $x(k+1) = \underline{A}x(k) + \underline{B}u(k)$ ) in desired locations, assuming the pair ( $\underline{A}, \underline{B}$ ) controllable.

### 3.2 Solution of Regulator equations

Except in very few cases, it is difficult to derive an analytical solution to the mappings  $x = \pi(\rho)$  and  $u = c(\rho)$  that solve the regulator equations, since it requires solving a set of complex non-linear difference equations (12). However, in practice, approximated solutions are acceptable [13]. Castillo *et al* [16] have presented conditions for the existence of an approximate solution to the non-linear discrete-time problem, based on a polynomial expansion. Similarly, Huang and Rugh [22] have presented for the continuous case an approximate solution based on Taylor series expansion of the plant zero-error manifold. Chu and Huang [23] have proposed a method based on a class of *RNN* to solve the regulator equations, which has the features of a cellular neural network. They have shown that under a proper choice of its parameters the recurrent network is capable of solving the non-linear differential equations.

The main contribution of this work is to propose, based on the particular *RNN* non-linearities (hyperbolic tangent), an approximate method that efficiently solves the regulator equations. The algorithm can be understood as a gradient method, such that the adaptation gain is variant and adjusted to guarantee the stability of the iterative procedure. The problem is driven to an eigenvalues assignment,

being the convergence guaranteed once appropriate eigenvalues are selected.

### 3.3 Proposed Algorithm

The solution of the regulator equation relies on the evaluation of the two mappings,  $x(k) = \pi(\rho(k))$  and  $u(k) = \alpha(\rho(k))$ , satisfying (12). Let us define the vector  $\Gamma(\rho(k)) \in \mathfrak{R}^m$ , composed of these mappings, whit  $m = n + n_u$ .

$$\Gamma(\rho(k)) = \begin{bmatrix} \pi(\rho(k)) \\ \alpha(\rho(k)) \end{bmatrix} \quad (14)$$

For clarity it is assumed the following notation  $\pi(\rho(k)) \in \mathfrak{R}^n = \pi(\rho)$  and  $\alpha(\rho(k)) \in \mathfrak{R}^{n_u} = \alpha(\rho)$ . It is considered a square system ( $n_y = n_u$ ) and a constant reference problem. The application of the zero output constrained dynamics algorithm [17] to the present problem leads to the following set of  $m$  equations:

$$\begin{aligned} A \pi(\rho) + B \alpha(\rho) + D \sigma(\pi(\rho)) &= \pi(\rho) \\ C \pi(\rho) &= r(\rho) \end{aligned} \quad (15)$$

Let us assume the existence of a *true* solution  $\pi(\rho)^*$  and  $\alpha(\rho)^*$  that verifies (16)

$$\begin{aligned} 0 &= A \pi(\rho)^* + B \alpha(\rho)^* + D \sigma(\pi(\rho)^*) - \pi(\rho)^* \\ 0 &= C \pi(\rho)^* - r(\rho) \end{aligned} \quad (16)$$

Given a solution  $\pi(\rho)^i$  and  $\alpha(\rho)^i$  for each iteration  $i$  one obtains the error  $E_1^i \in \mathfrak{R}^n$  and  $E_2^i \in \mathfrak{R}^{n_u}$ , defined by (17).

$$\begin{aligned} E_1^i(k) &= [ A \pi(\rho)^* + B \alpha(\rho)^* + D \sigma(\pi(\rho)^*) - \pi(\rho)^* ] \\ &\quad - [ A \pi(\rho)^i + B \alpha(\rho)^i + D \sigma(\pi(\rho)^i) - \pi(\rho)^i ] \\ E_2^i(k) &= [ C \pi(\rho)^* - r(\rho) ] - [ C \pi(\rho)^i - r(\rho) ] \end{aligned} \quad (17)$$

Let us also define the variation:

$$\Gamma(\rho)^{i+1} = \begin{bmatrix} \pi(\rho)^{i+1} \\ \alpha(\rho)^{i+1} \end{bmatrix} = \begin{bmatrix} \pi(\rho)^i \\ \alpha(\rho)^i \end{bmatrix} + \begin{bmatrix} \Delta \pi(\rho)^i \\ \Delta \alpha(\rho)^i \end{bmatrix} \quad (18)$$

From(17) follows the variation error between two consecutive iterations  $i + 1$  and  $i$ , (19).

$$\begin{aligned} E_1^{i+1}(k) - E_1^i(k) &= -A \Delta \pi(\rho)^i - B \Delta \alpha(\rho)^i \\ &\quad - D [ \sigma(\pi(\rho)^i + \Delta \pi(\rho)^i) - \sigma(\pi(\rho)^i) ] + \Delta \pi(\rho)^i \\ E_2^{i+1}(k) - E_2^i(k) &= -C \Delta \pi(\rho)^i \end{aligned} \quad (19)$$

Since the non-linearity (hyperbolic tangent) is Lipschitz, one can write (20)

$$\| \sigma(\pi(\rho) + \Delta \pi(\rho)) \| \leq \| \sigma(\pi(\rho)) \| + \| \Delta \pi(\rho) \| \quad (20)$$

or approximately (21)

$$\sigma(\pi(\rho) + \Delta \pi(\rho)) \approx \sigma(\pi(\rho)) + G \Delta \pi(\rho) \quad (21)$$

being the matrix  $G \in \mathfrak{R}^{n,n}$  diagonal defined by (22).

$$G = \text{diag} \{ \sigma'(\pi(\rho)) \} \quad (22)$$

Equation (17) can be written as:

$$E^{i+1}(k) - E^i(k) = -M \Delta \Gamma(\rho)^i \quad (23)$$

where  $E^i \in \mathfrak{R}^m$  is the error corresponding to the mappings  $\pi(\rho)^i$  and  $\alpha(\rho)^i$  at each iteration  $i$ .  $I_n \in \mathfrak{R}^{n,n}$  is an identity matrix and  $M \in \mathfrak{R}^{m,m}$  is a matrix evaluated from the original system matrices.

$$M = - \begin{bmatrix} A + D G - I_n & B \\ C & 0 \end{bmatrix} \quad (24)$$

Following this approach the problem can be stated as a pole placement one. Actually, if the variation  $\Delta \Gamma(\rho)^i$  is specified by (25)

$$\Delta \Gamma(\rho)^i = F E^i(k) \quad (25)$$

one obtains (26),

$$E^{i+1}(k) = [ I_m - F M ] E^i(k) = \Lambda E^i(k) \quad (26)$$

where  $I_m \in \mathfrak{R}^{m,m}$  and  $F \in \mathfrak{R}^{m,m}$ . Thus, it is possible to assure the error convergence if the matrix  $F$  is adequately chosen such that  $\Lambda \in \mathfrak{R}^{m,m}$  is a Hurwitz matrix.

**Theorem:** Given the system (2), if the matrix  $F$  is determined such that the eigenvalues of the matrix  $\Lambda$  are placed inside the unitary circle, then the following specification for the variation  $\Delta \Gamma$

$$\Delta \Gamma(\rho)^i = F E^i(k) \quad (27)$$

guarantees the error convergence, if

- i) the pair  $(A + DG, B)$  controllable
- ii)  $\text{rank}(M) = m$

#### Proof

In order to apply the *CMT*, that assures the existence of a solution for the regulator equations, some conditions have to be verified [16]. The first requires a hyperbolic stable reference which is automatically verified since it is assumed a constant reference. The second requirement imposes the controllability condition for the first approximation of system, given by condition *i*). On the other hand, the existence of a solution to the regulation problem is related with the error convergence. However, this problem can be understood as a state space feedback control. In fact, considering the common notation (28) for this problems

$$x(k+1) = \underline{A} x(k) + \underline{B} u(k) \quad (28)$$

it is possible to relate (23) with (28), as follows:  $x(k) = E^i(k)$ ,  $\underline{A} = I_m$ ,  $\underline{B} = -M$ ,  $u(k) = \Delta \Gamma(\rho)^i$ . It is known, from the linear control theory, that if system (28) is controllable then it is possible, by means of a state space feedback, to find a control law  $u(k) = F x(k)$  such that  $\lim_{k \rightarrow \infty} x(k) = 0$ . Furthermore,  $F$  exists if the controllability condition (29) for the pair  $(\underline{A}, \underline{B})$ , is verified,

$$\text{rank} \left( \begin{bmatrix} \underline{B} & \underline{A} \underline{B} & \underline{A}^2 \underline{B} & \dots & \underline{A}^{n-1} \underline{B} \end{bmatrix} \right) = n \quad (29)$$

Analogous to (29) it should be verified (30).

$$\text{rank} \left( \begin{bmatrix} -M & -IM & \dots & -I^{m-1}M \end{bmatrix} \right) = m \quad (30)$$

which is equivalent to

$$\text{rank}(M) = m \quad (31)$$

■

## 4. APPLICATION TO A CSTR

Among several works that propose the application of neural networks to *CSTR* one can highlight the works of Lightbody and Irwin [24], Ge *et al* [25] and Wei [26].

### 4.1 Modeling the CSTR process

The *CSTR* consists of a constant volume reactor  $V$ . The variables  $Q_a$ ,  $C_{ai}$  and  $T_{ai}$  define, respectively, the flow, concentration and temperature of the component to be concentrated.  $Q_c$  and  $T_{ci}$  define the flow and temperature of the coolant stream that flows in a concurrent fashion, as shown in Figure 1.

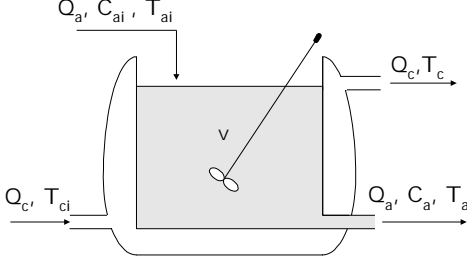


Figure 1: Continuous Stirred Tank Reactor.

At the tank output the variables under consideration are the temperature of the coolant flow  $T_c$ ,  $C_a$  and  $T_a$ , respectively, the concentration and temperature of the concentrated component. The objective is to set the effluent concentration  $C_a$  to a set-point value  $y_d$  by manipulating the coolant flow rate  $Q_c$ . The process model here described was proposed by [24], defined by (32) and (33).

$$\dot{C}_a(t) = (C_{ai} - C_a(t)) - r_a(t) \quad (32)$$

$$\begin{aligned} \dot{T}_a(t) = & \frac{Q_{ai}}{V} (T_{ai} - T_a(t)) + k_1 C_a(t) e^{\frac{-E}{RT_a(t)}} \\ & + k_2 Q_c(t) (1 - e^{\frac{-k_3}{Q_c(t)}}) (T_{ci} - T_a(t)) \end{aligned} \quad (33)$$

The variable  $r_a(t)$  defines the rate per unit volume of component, given by  $r_a(t) = \kappa C_a(t)$ , being  $\kappa$  the reaction rate coefficient component,  $E$  the activation energy,  $R$  the gas constant,  $k_1$  and  $k_2$  are constants [24]. Typically,  $\kappa$  is a strong function of reaction temperature and can be described by the Arrhenius relation [24]. From the analysis of the equations describing the process, it can be observed that the non-linear characteristics of the process are mainly due to the presence of the exponential terms. The considered equilibrium point is:  $Q_{co} = 103.41$  l/min,  $C_{ao} = 0.1$  mol/l and  $T_{ao} = 438.54$  K.

To obtain an initial estimation for the network parameters a number of test inputs were applied. The goal in designing the test inputs was to cover the operational range of the plant as extended as possible. The number of training patterns, hidden neurons, and input sequence are all chosen by experiments since there is still no reliable method of determining these parameters systematically and automatically.

It was found that a selection of four hidden neurons,  $n = 4$ , is suitable to obtain a good model for the *CSTR* process. The Levenberg-Marquardt algorithm was applied allowing to obtain an initial value for the matrices  $A$ ,  $B$ ,  $C$ , and  $D$  defined in (2). During operation the matrices  $A$  and  $C$  are kept fixed and only  $B$  and  $D$  are updated according to (5).

### 4.2 Simulations Results

Several simulations were conducted to test the effectiveness and adaptivity of the neural network based controller for the *CSTR*. The sampling time was chosen  $T=0.25$  minutes and a 4<sup>th</sup> order Range-Kutta method was used.

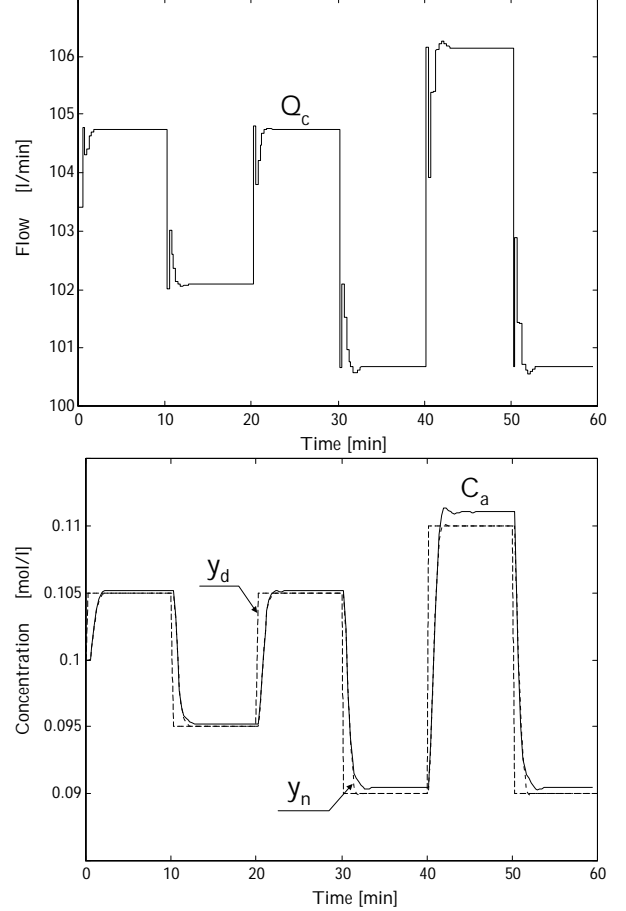


Figure 2. Fixed non-linear controller.

Figure 2 shows the behavior using a fixed non-linear controller based on the *a-priori* determined recurrent neural model. As can be seen, the dynamical response is adequate, however there are control errors, perfectly justified due to the inaccuracy of the initial neural model.

In the second simulation the parameters are adjusted on-line, resulting in an adaptive control scheme. The control systems' response is shown in Figure 3. This control scheme, by on-line adjusting the model parameters, enables to reduce gradually model plant mismatches and by this way contributes to the convergence of steady-state offsets to zero.

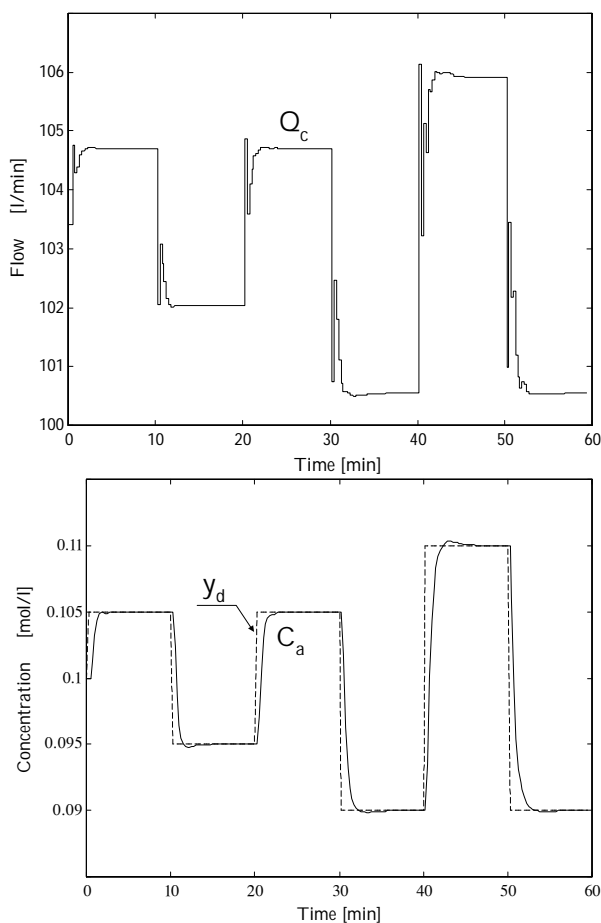


Figure 3. Adaptive non-linear controller.

## 5. CONCLUSIONS

In this work a methodology for analysis and design of a non-linear controller is proposed. It is based on the output regulation theory, based on a recurrent neural network. The control objectives specified by using a recurrent neural model requires the controller to be a non-linear one. An iterative technique that solves the regulation problem was proposed, shown to be stable in the sense of Lyapunov. The feasibility of the proposed methodology is tested on a benchmark problem, the continuous stirred tank reactor. This study has shown that neural networks are an important paradigm for many control applications, specially when the processes are non-linear. The simplicity and reliability of neuro-control schemes over traditional control techniques will be the key for the development of efficient and intelligent control systems in the near future.

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