SLS Algorithms for Multiobjective Combinatorial Optimization

Luís Paquete

CISUC, Department of Informatics Engineering,
Faculty of Science and Technology,
University of Coimbra,
Coimbra, Portugal

paquete@dei.uc.pt

Topics

1. Introduction
2. MCOPs and solution methods
3. SLS algorithms
4. Performance assessment
5. Further work

Multiobjective Combinatorial Optimization Problems (MCOPs)

- Many real-life problems are multiobjective
  - Telecommunications and computer networks
  - Logistics and transportation
  - Timetabling and scheduling
  - ... and many others

- But most MCOPs are NP-hard and intractable

How to design and analyze SLS algorithms for MCOPs?

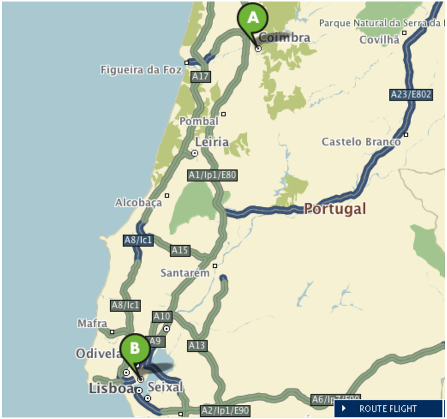
From Coimbra to Lisbon by car

The fastest path:

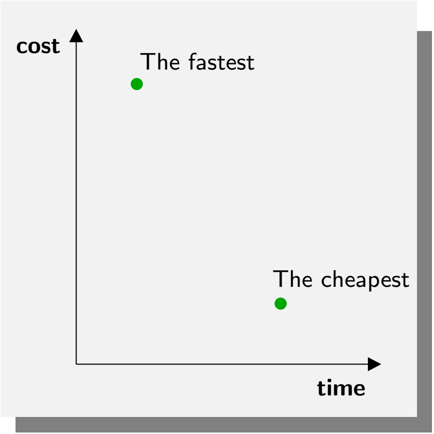
- 1h 53m and 30 €

The cheapest path:

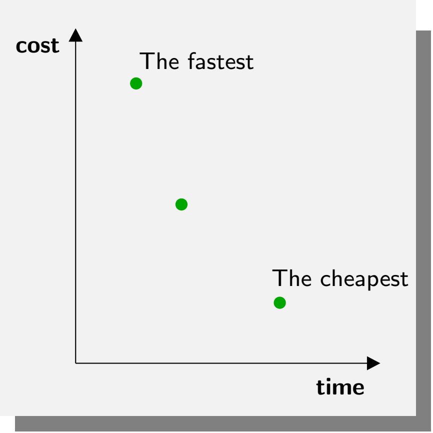
- 3h 05m and 13 €



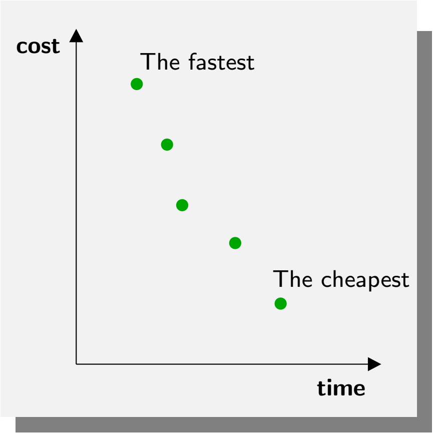
Introduction



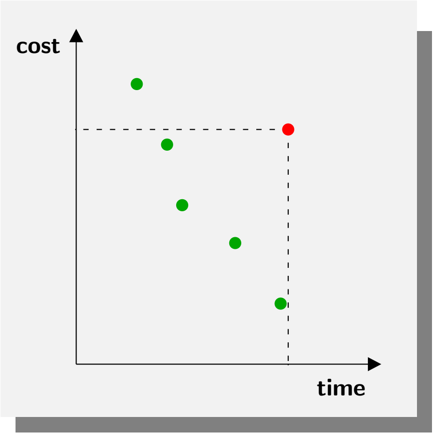
Introduction

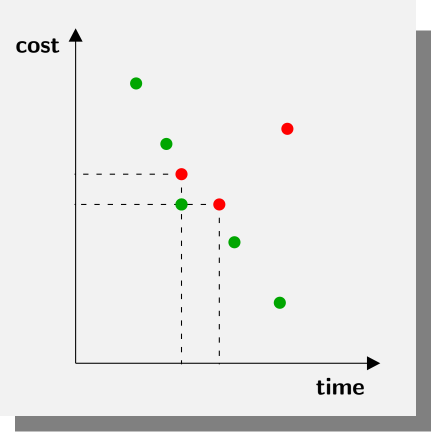


Introduction



Introduction




cost
time

Multiobjective Combinatorial Optimization Problem

The set X of feasible solutions is finite and its elements have some combinatorial property (graph, tree, path, partition, etc.).

The goal is to

$$\min_{x \in X} \mathbf{f}(x) = (f_1(x), \ldots, f_Q(x))$$

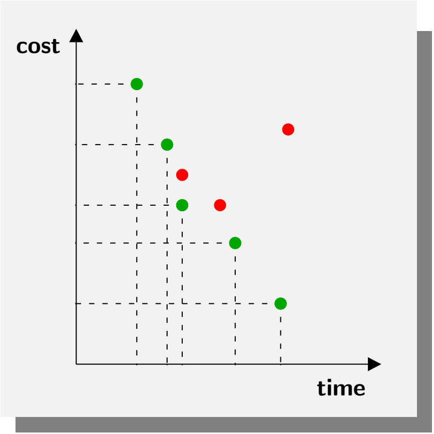
- The objective function $\mathbf{f}$ maps $x \in X$ to $\mathbb{R}^Q$

- Optimality depends of the decision maker's preferences (or lack of them).
- Pareto-optimality is based on component-wise order :

$$\mathbf{u} \leq \mathbf{v} \iff \mathbf{u} \neq \mathbf{v} \text{ and } u_i \leq v_i,\ i = 1, \ldots, Q$$

- A solution $x \in X$ is efficient iff $\nexists\, x' \in X$ s.t. $\mathbf{f}(x') \leq \mathbf{f}(x)$
- Efficient set is the set of all efficient solutions
- Nondominated set is the image of the efficient set in $\mathbf{f}$

Introduction



- Most MCOPs are NP-hard

Decision version of MCOP (MCOP-D) [Serafini 1986]:
Given $\mathbf{z} = (z_1, \ldots, v_Q)$, does there exist a solution $x \in X$ s.t.

$$\mathbf{f}(x) \leq \mathbf{z} \;\text{ or }\; \mathbf{f}(x) = \mathbf{z}?$$

1. If the single-objective problem is NP-complete, then the corresponding MCOP-D is also NP-complete.

2. If the single-objective problem is solvable in polynomial time, the corresponding MCOP-D may still be NP-complete.

- Biobjective Shortest Path Problem (decision version):

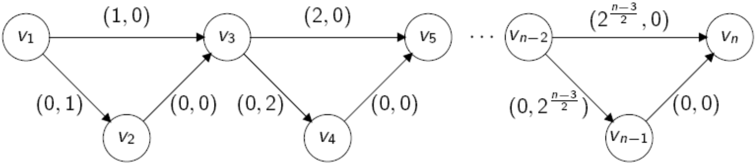
  Instance: A graph $G = (V, E)$, distance $d(e) \in \mathbb{Z}^+$ and cost $c(e) \in \mathbb{Z}^+$ for each $e \in E$, specified nodes $s, t \in V$, and $D, C \in \mathbb{Z}^+$.

  Question: Is there a path in G from s to t with total distance D or less and total cost C or less?

- It's also the Shortest Weight-Constrained Path Problem [Garey & Johnson 1979].

- Then, the Biobjective Shortest Path Problem is NP-complete [Hansen 1979].

- There may be an intractable number of efficient solutions.



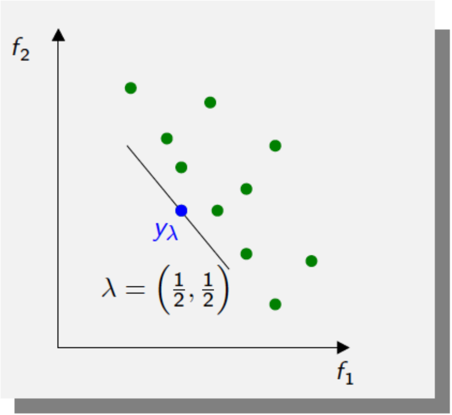
There exist $2^{\frac{n-1}{2}}$ efficient paths [Hansen 1979].

Solution Methods to MCOPs

- Enumeration Methods
  - Multiobjective Branch & Bound
  - Multiobjective Dynamic Programming
- Scalarized Methods
  - Solving several related single-objective problems
  - Weighted Sum, Compromise Programming, ϵ-constraint, etc.
- SLS Algorithms

Weighted Sum

- $\min_{x \in X} \sum_{i=1}^{Q} \lambda_i f_i(x)$
- λ gives a search direction
- An optimal solution with $\lambda > 0$ is efficient.



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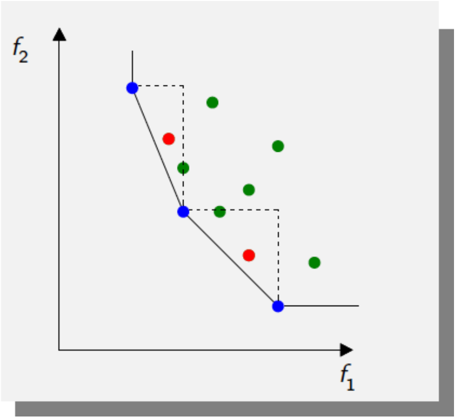
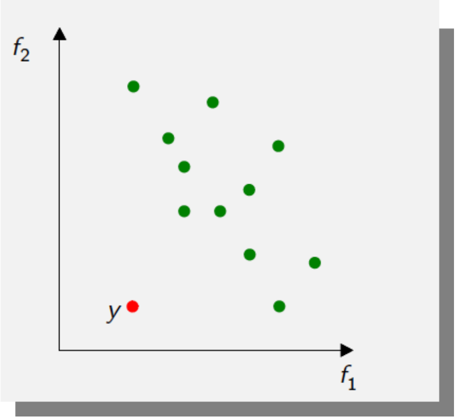


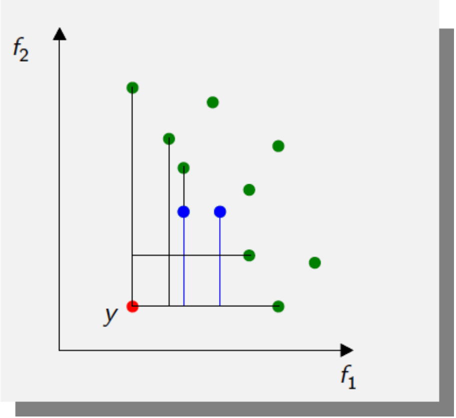
Weighted Compromise Programming

- $\min_{x \in X} d\left(\mathbf{f}(x), y\right)$
- Efficiency depends of the distance d
- d is derived from L^p norms



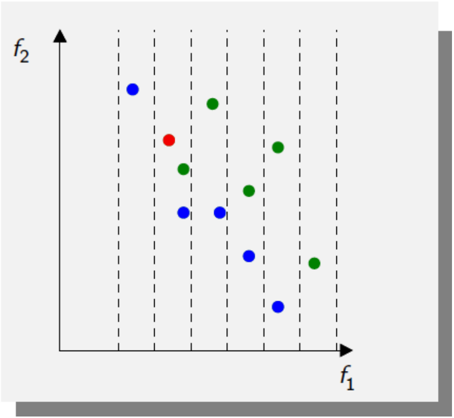
Weighted Compromise Programming

- $\min_{x \in X} \max_{i=1,\ldots,Q} (\lambda_i |f_i(x) - y_i|)$
- An optimal solution with $\lambda > 0$ is weakly-efficient



ϵ−constraint

- $\min_{x \in X} f_2(x) : f_1(x) \leq \epsilon$
- An optimal solution for an ϵ is weakly-efficient



Total enumeration

- If the feasible set X is small, enumerate it and solve a minima-finding problem in $Z = \mathbf{f}(X)$
  - $O\left(|Z|^2\right)$ for $Q \geq 2$
  - $O\left(|Z| \cdot \log |Z|\right)$ for $Q = 2$ [Kung et al. 1975]
  - $O\left(|Z| \cdot (\log |Z|)^{Q-2}\right)$ for $Q = \{3, 4, 5\}$ [Kung et al. 1975]

SLS Algorithm design challenges for MCOPs

- How to attain more than one solution?
- How to attain high quality solutions?
- How to evaluate performance?

Rule of thumb

- Closeness to the nondominated set
- Well-distributed outcomes
- The more, the better

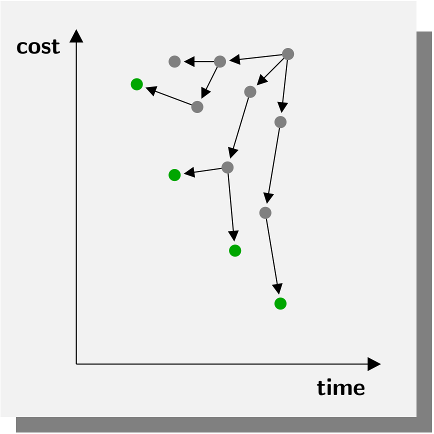
Multiobjective Local Search

input:candidate solution x
while x is not a local optimum **do**
  choose a neighbor x' from x such that $\mathbf{f}(x') \leq \mathbf{f}(x)$
  $x = x'$
return x

- What if $f(x')$ and $f(x)$ are mutually nondominated?
- How to obtain more than a single solution?

CWAC Search Model

input: canditate solution x
Add x to Archive
repeat
  Choose x from Archive
  $X_N = \text{Neighbors}(x)$
  Add X_N to Archive
  Filter Archive
until all x in Archive are *visited*
return Archive



CWAC Search Model

input: canditate solution x, ϵ
Add x to Archive
repeat
  Choose x from Archive
  X_N = Neighbors(x)
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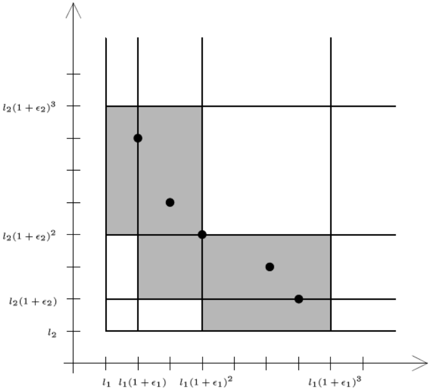

Archive bounding
[Angel et al. 2004]

CWAC Search Model

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  Filter Archive according to ϵ
until all x in Archive are *visited*
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- Search Strategy
- Archive Bounding
- Neighborhood

Scalarized Acceptance Criterion (SAC) Model

- Weighted Sum

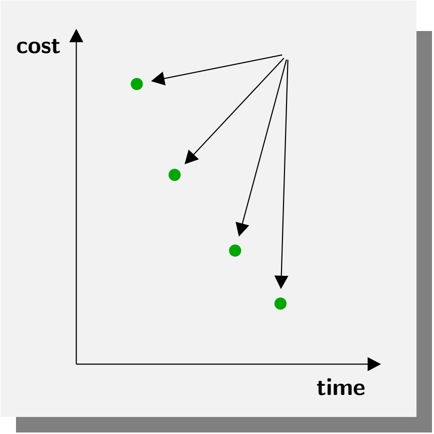
$$f(x) = \sum_{i=1}^{Q} \lambda_i f_i(x)$$

- Weighted Chebycheff

$$f(x) = \max_{i=1,\dots,Q} \left(\lambda_i \mid f_i(x) - y_i \mid \right)$$

SAC Search Model

input: weight vectors Λ
for each $\lambda \in \Lambda$ **do**
  x is a candidate solution
  $x' =$ SolveSAC(x, λ)
  Add x' to Archive
Filter Archive
return Archive



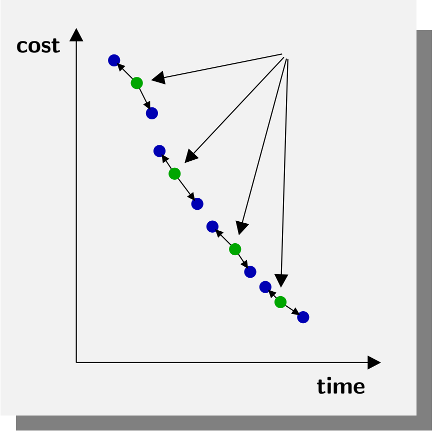
SAC Search Model

input: weight vectors Λ
for each $\lambda \in \Lambda$ **do**
  x is a candidate solution
  $x' = \text{SolveSAC}(x, \lambda)$
  Add x' to Archive
Filter Archive
return Archive

- Search Strategy
- Number of Scalarizations
- Intensification Mechanism
- Neighborhood

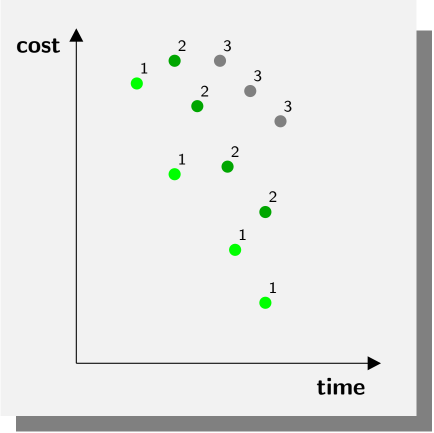
Hybrid Search Model

input: weight vectors Λ
for each $\lambda \in \Lambda$ **do**
  x is a candidate solution
  $x' = \text{SolveSAC}(x, \lambda)$
  $X' = \text{CW}(x')$
  Add X' to Archive
Filter Archive
return Archive



SAC Search Model – EMO

input: canditate solution set X_n
repeat
  X_r = Reproduce/Mutate(X_n)
  R = Rank(X_r, X_n)
  X_s = Select(X_r, X_n, R)
  X_n = Replace(X_s)
return X_n



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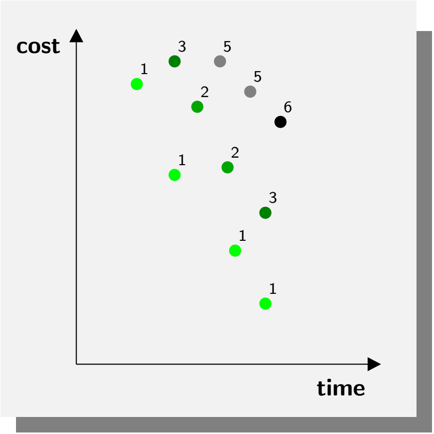

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$X_n = \text{Replace}(X_s)$
return X_n

- Component-wise order
- Closeness
- Performance indicators

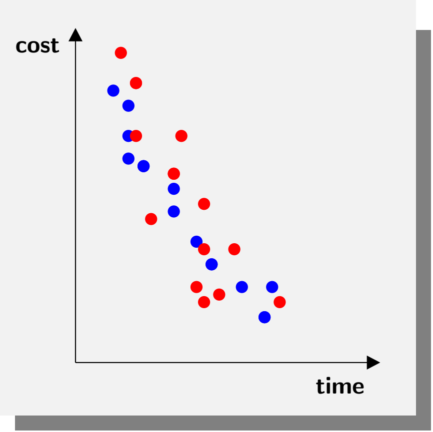
Rules of Thumb: An algorithm performs better if

- It is closer to the nondominated set
- It has better distributed outcomes
- It has more solutions

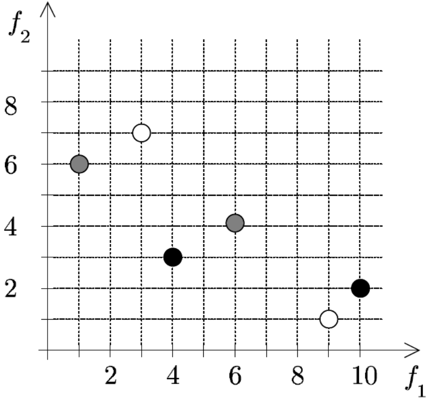
Indicators of Performance

- Measure some property of the outcomes
- Most of the indicators have limitations
  [Knowles & Corne 2002, Zitzler et al. 2003]

Performance Assessment

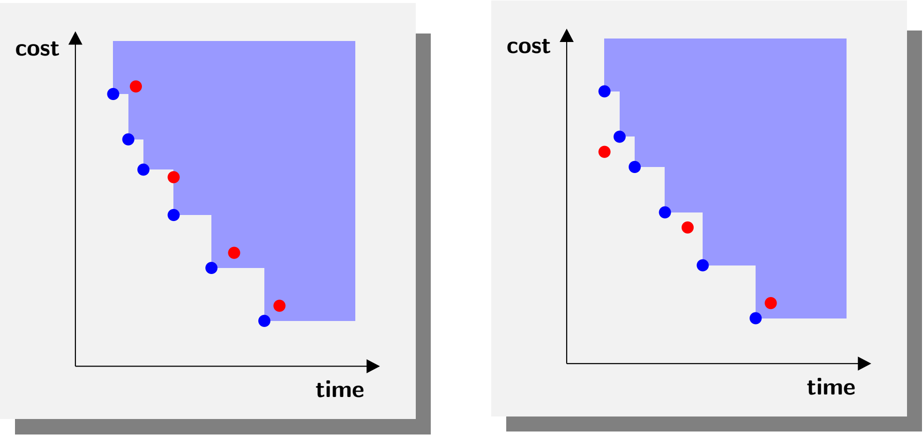


Many runs of Algorithms Blue and Red



Another example

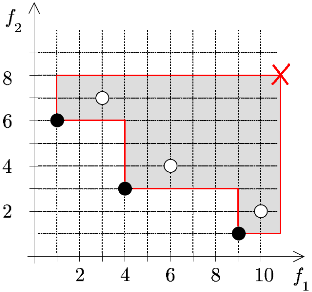
- Better relations
  [Hansen & Jaszkiewicz 1998, Zitzler et al. 2003]



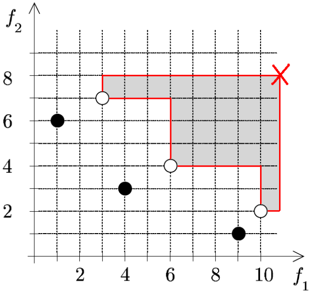
Blue is better than Red

Blue and Red are incomparable

- Unary Indicator: Hypervolume
  [Zitzler and Thiele, 1998]



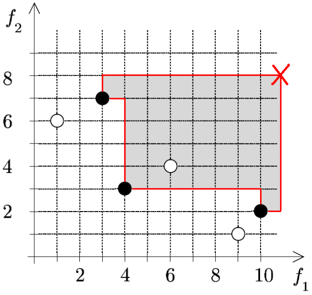
$H(B) = 45$



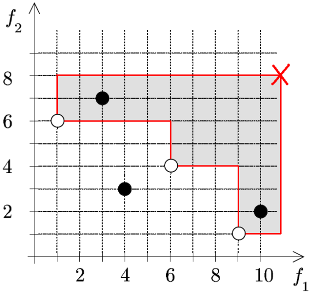
$H(W) = 25$

B is better than W $\Longrightarrow$ $H(B) > H(W)$

- Unary Indicator: Hypervolume
  [Zitzler and Thiele, 1998]



$H(B) = 37$

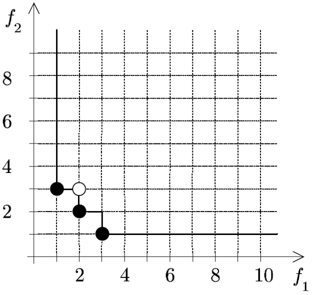


$H(W) = 36$

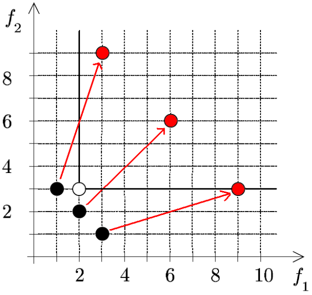
$H(B) > H(W) \implies B$ is not worse than W

- Binary Indicator: Binary ϵ-indicator [Zitzler et al., 2003]

$$I_\epsilon(A, B) = \max_{b \in B} \min_{a \in A} \max_{i=1,\ldots,Q} \frac{a_i}{b_i}$$



$I_\epsilon(B, W) = 1$

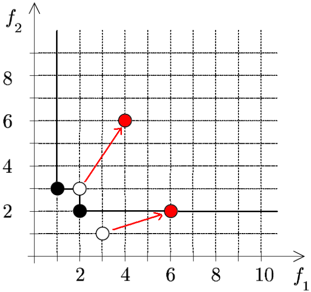


$I_\epsilon(W, B) = 3$

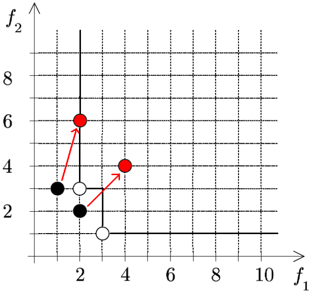
$I_\epsilon(B, W) \leq 1 \quad \wedge \quad I_\epsilon(W, B) > 1 \quad \Longleftrightarrow \quad B$ is better than W

- Binary Indicator: Binary ϵ-indicator [Zitzler et al., 2003]

$$I_\epsilon(A,B) = \max_{b\in B} \min_{a\in A} \max_{i=1,\dots,Q} \frac{a_i}{b_i}$$



$I_\epsilon(B,W) = 2$

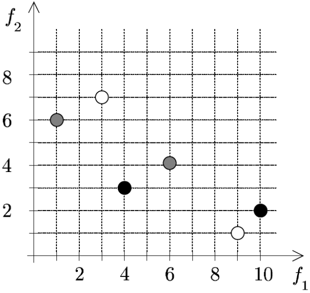


$I_\epsilon(W,B) = 2$

$I_\epsilon(B,W) > 1 \quad \wedge \quad I_\epsilon(W,B) > 1 \quad \Longleftrightarrow \quad W$ and B are incomparable

- Binary Indicator: Binary ϵ-indicator [Zitzler et al., 2003]

$$I_\epsilon(A, B) = \max_{b \in B} \min_{a \in A} \max_{i=1,\dots,Q} \frac{a_i}{b_i}$$



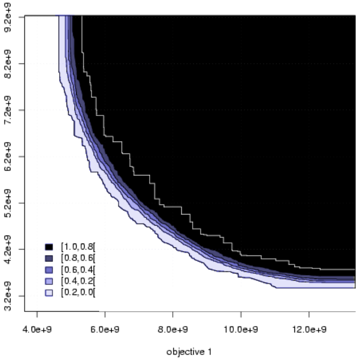
The larger value, the better?

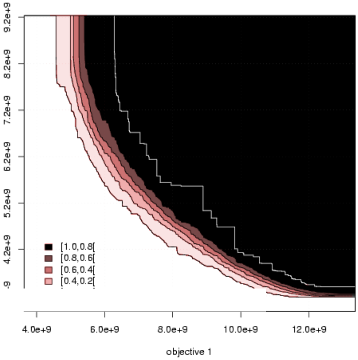
$I_\epsilon(c, r)$	White	Gray	Black
White	-	<u>4.0</u>	2.0
Gray	3.0	-	<u>4.0</u>
Black	<u>2.25</u>	2.0	-

- Attainment Functions [V.G. da Fonseca et al. 2001]

  AF: Prob. that an outcome set is better or equal to z.

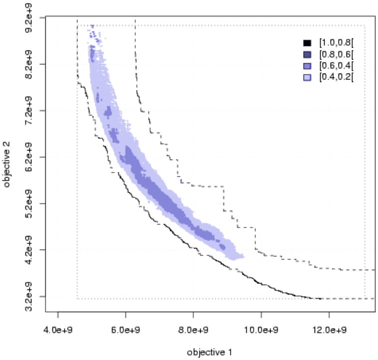
  EAF: How many runs an outcome set is better or equal to z?



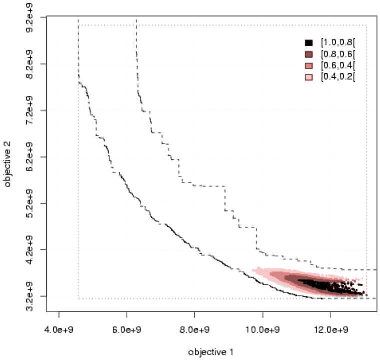


- Attainment functions – Visualization of differences

$$EAF_{Blue} - EAF_{Red}$$



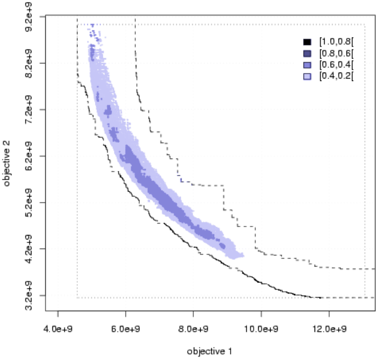
positive differences



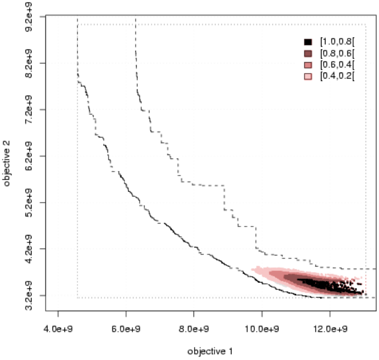
negative differences

- Attainment functions – Statistical testing

K-S test statistic: $\max \mid EAF_{Blue} - EAF_{Red} \mid$



positive differences



negative differences

- Diversity and sharing techniques in EMO.
- ACO, GRASP, PSO, Scatter Search, Path Relinking.
- Interaction with exact approaches and CP.
- Connectedness of efficient solutions.
- Statistical properties of fitness landscapes in MCOPs.

Further work

- Test in hard and real-life problems.
- Understand the effects and interactions between SLS components.
- Role of instance features on performance of SLS algorithm.
- Mathematical relation between performance indicators.

References

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- Complexity and Approximation: P. Hansen 1979, P. Serafini 1986, M. Ehrgott 2000, C.H. Papadimitriou and M. Yannakakis 2000, E. Angel et al. 2007.
- Performance Assessment: E. Zitzler et al. 2003, 2008, V.G. da Fonseca et al. 2001, 2010, M. Lopéz-Ibáñez et al. 2010.
- Web material: PISA (`http://www.tik.ethz.ch/~sop/pisa`), MOMH (`http://home.gna.org/momh`), ParadisEO (`http://paradiseo.gforge.inria.fr`)